



Minimal Dimensions of a Small House by Means of Optimization Procedure

J. Horta Rangel^{1*}, Gómez Ruiz Aimé¹, L. Pérez Rea¹, E. Rojas González¹,
T. López Lara¹ and J. Hernandez Zaragoza¹

¹Department of Graduate Engineering, Universidad Autónoma de Querétaro, Querétaro, Qro., 76010, México.

Authors' contributions

This work was carried out in collaboration between all authors. Authors JHR and GRA have designed the study and managed the analyses. Authors TLL and JHZ have conducted the literature searches, Authors LPR and ERG has reviewed and improved the final manuscript. All authors read and approved the final manuscript.

Article Information

DOI: 10.9734/BJAST/2015/18170

Editor(s):

(1) Jakub Kostecki, Department of Civil and Environmental Engineering, University of Zielona Góra, Poland.

Reviewers:

- (1) Anonymous, Shaanxi Normal University, China.
(2) Anonymous, Kaunas University of Technology (KTU), Lithuania.
(3) Anonymous, University of Trieste, Trieste, Italy.
(4) Massimo Palme, School of Architecture, Catholic University of the North, Chile.
Complete Peer review History: <http://sciencedomain.org/review-history/10342>

Original Research Article

Received 8th April 2015
Accepted 21st July 2015
Published 28th July 2015

ABSTRACT

This work has focused on the analysis of minimum dimensions of housing while its occupants maintain a proper comfort within it according to several requirements: proxemics, ergonomics, anthropometry as well maintaining an energy efficiency of the house. The current analysis is referred to a small house with four occupants and according to Mexican standards. These constructions are built in a large number to satisfy needs society. Optimal design of architectural spaces of a small house makes sense in that housing should ensure a minimum of comfort for its occupants, maintaining minimum and standard requirements. However, current constructive trends of these houses tend to minimize its costs based mainly on two policies: reduced construction areas and use of inappropriate quality materials. On this work it was proposed an optimization procedure for finding the optimal dimensions of a small house through a mathematical model that takes the overall volume of housing as an objective function, while the design variables correspond to the

*Corresponding author: E-mail: jaimhorta@prodigy.net.mx;

main dimensions of the house while state equations correspond to the requirements characterizing all living spaces including thermal comfort and home energy costs.

Keywords: Optimal housing design; proxemic; energy efficiency; feasible optimal solutions.

1. INTRODUCTION

The review of the optimal spaces in a home is a topic of great interest and mainly in the fields of architecture as well as in engineering construction. The architectural design has a wide range of possibilities and depends on the creativity of the architect. The house design depends on the particular family needs and the requirements and circumstances of their environment [1]. The particular case of small housings for low-income people is a major issue from several perspectives: social, economic, and even political. The massive construction of housing in many countries tends to homogenize and reduce the areas of small housing, in part due to the aim of reducing costs and production housing request.

The United Nations (UN) established in the International Declaration on Economic, Social and Cultural Rights, the right of everyone to an adequate standard of living for himself and his family, including an appropriate housing, which includes thermo-acoustic properties as well as an adequate orientation, identifying the minimum spaces for the all required activities within the home and coexistence among users from proxemic point of view [2]. The construction procedure of living homes, building materials, its architectural design, etc. must appropriately enable the expression of cultural identity and diversity of housings. Thus, individuals and their families feel part of a community with shared cultural values, traditions and a close lifestyle.

Optimization Procedures applied in housing has been mainly developed in the last two decades. Early there were developed optimal solutions for buildings and industrial constructions but focused on their structural behavior together with their minimal weight and cost [3]. One of the applications more referred in the literature relates to air conditioning optimal solution in buildings; the optimization problem based on housing is still a current research topic. The optimization algorithms consist of minimize or maximize a function which is described through some independent parameters named designer variables. Depending on the values conferred to

those variables will be the value of such a function known as objective function. The state variables consist of any conditions or requirements imposed to the objective function [4].

There are mathematical procedures to evaluate the minimum of a function under the restrictions requested; procedures are generally non-linear because of the structure of the equations. In some cases, when the number of variables are small, geometric solution becomes a solution by making a projection of graphs with different variable values which allow further display the area of feasible solutions, this kind of procedure is used only when the number of independent variables is less than or equal to three.

The architectural design of housing requires a distribution of spaces inside it based on the movements of people within the housing. Each space: living room, kitchen, bedroom, bathroom, requires appropriate dimensions to move inside it. Those spaces include the furniture and equipment for the proper performance of all their duties. On this work we focus the attention on a the optimization of a small housing developing the optimization procedure by building an objective function and state functions taking into account the main three variables of housing area: Its length L , width A , and height H . Also, according to the Mexican National Institute of Statistics and Geography (INEGI) on a small housing lives four member [5]; in our work this is a condition to carry on the optimization procedure.

2. METHODOLOGY

Anthropometry is the study of human body measurements in all positions and activities, is the ratio of the dimensions of a man and space required to move and to be comfortable in different positions [6]. Although the human body dimensions cannot be generalized, there are statistics data that provide the dimensions depending on depending on where the person is living, in this research was selected a latin american man with a main height of 1.73 meters [7]. Moreover ergonomics is the set of scientific

knowledge applied to work, systems, products and environments, to adapt to the physical and mental abilities as well to the limitations of people. It is important to understand that even though the design and style of the person inhabiting, differ among themselves. Some aspects may be standardized, as an example, Fig. 1 shows the use of an individual basin, where the space required for the use of furniture without restricting all his movements, is specified.

In this research has been incorporated the Proxemics as it provides the distances that surround a one person, that means to determine the spaces (dimensions) that should be maintained while individuals are interacting with each other's . This space not only includes the physical space properly, but should be seen as the required space needed by the individuals that will influence somehow on his physical and mental behavior. On housings with small spaces and with many family members become of the great importance the privacy and the free movement in the available space [8]. It has been

defined four spaces radius: the public space, the social ones, the personal and the intimate space. On this work we have considered the personal space, taking a distance from 46 to 120 centimeters meanwhile the intimate space from 15 to 45 centimeters.

Housings must save mankind from climate and provide an environment that will facilitate wellness and performance of their activities. To achieve this it is necessary to have a clean and healthy air periodically renewing it. The respiration of the individuals, result in consumption of oxygen and expulsion of carbon dioxide [1]. This process in turn produces an emission of 0.020 m³ / h of carbon dioxide (CO₂) and 40 g/h water vapor in varying amounts depending on the activity. The air of a room may not contain more than 1% of CO₂, which means if there is a renewal of air every hour of air 32 m³ per adult and 15 m³ per child, are usually sufficient take 25 m³ of air per individuals inhabiting housing [9].

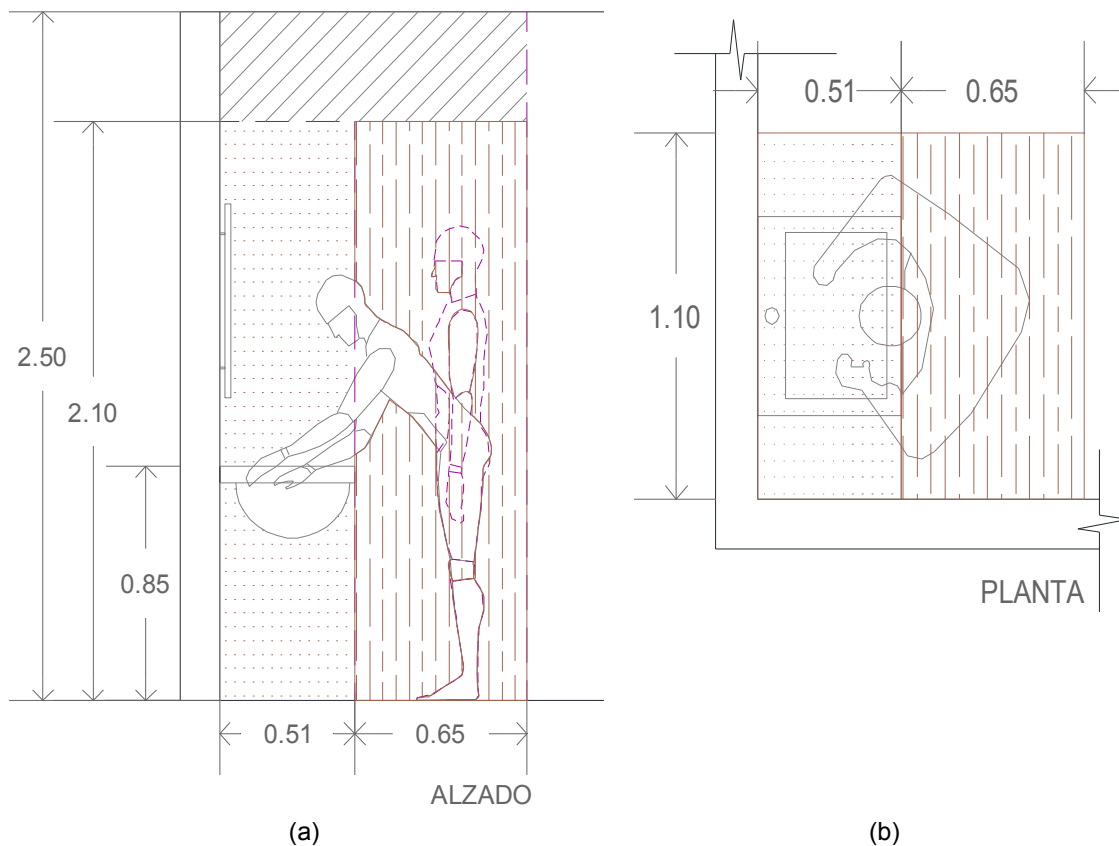


Fig. 1. Anthropometry and ergonomics applied to define minimal dimensions for the basin spaces. (a) front view, (b) plan view. fonseca X. [6]. unit (m)

3. ENERGY EFFICIENCY

The research and focus for energy efficiency in homes represents an effort to improve the thermal design and achieve the comfort of their occupants with minimum energy consumption. Since 1984 the World Commission of Environment and Development (WCED) has associated the development and environment in a single concept of "sustainable development". On this direction, Mexico has established the Standard NMX-C-460-2009 ONNCCE [10] with the purpose of contributing with investors, designers and builders to decrease the energy usage in housings mainly for air conditioning equipment, but also depending on climatic characteristics of each zone of the country, instead promote the use of thermal resistance materials for the envelopes of the housing (mainly walls and roof), determining thermal resistance R values of used local materials including insulation layers that in turn generate an envelopes with R values according Standards.

The minimum thermal resistance suggested by the standard ONNCCE for the area named 3A which belongs to the Querétaro state in Mexico, is $1.4 \text{ m}^2 \text{ }^\circ\text{K /W}$ for roofs while for walls is required a value of $1.0 \text{ m}^2 \text{ }^\circ\text{K/W}$. The Mexican Standard NOM-020-2011-ENER establishes the appropriate energy efficiency on buildings through the envelopes and the gain heat by conduction, as well the heat gain by solar radiation and the difference in temperatures inside and outside according to the orientation of the housing [11].

Based on the number of individuals within housing and in accordance to the demand of construction, we have developed the optimization functions related to the topics described above, as anthropometry, ergonomics, proxemics, and so on. Now, we will show the optimal procedure based mainly on the three variables referred to the overall dimensions of the housing. To analyse the energy efficiency of the home and according to the Mexican standard, a benchmark building is proposed. This reference building is analogous to the analysed housing, but including Standard recommendations, the goal of this procedure is to review the energy efficiency of the real house.

Previous analysis performed on real housing show that under Mexican Standard the housing envelopes do not meet the required minimal thermal resistance; therefore at this stage we

proceed previously to modify the materials of envelopes particularly on the walls. We added here a flat layer of expanded polystyrene (EPS) to the original conformation of the walls based on concrete blocks, thickness of EPS we obtain as 2.5 cm. On the other hand, in order to improve the thermal conductivity of the roof has proposed to change the constructive system by substituting concrete longitudinal block by bands made of EPS with a thickness of 7 cm. Thus, the minimum requirements for thermal resistance are satisfied. The above proposals allow us to build the state equations that will take part in the optimization model.

4. OPTIMIZATION MODEL

We require minimizing an objective function F corresponding to the whole volume of the house:

$$\min F(x) = f(x_1, x_2, x_3) \quad (1)$$

So, the state equations or restrictions are satisfied:

$$\begin{aligned} g_i(x) &= a_i \\ \dots\dots\dots \\ g_j(x) &\leq a_j \\ g_k(x) &\leq a_k \end{aligned} \quad (2)$$

At the same time, the design variables have a range min-max: $b_i \leq x_i \leq c_i$

Our case is referred to a rectangular small house, the objective function can be written as follow:

$$\min F(x) = L \cdot A \cdot H \quad (3)$$

Here: L is the length of the house; A is de width, and H their height. Unit: meters

The ranges of variation of the independent variables have been adopted as follows:

$$\begin{aligned} 7.0 &\leq L \leq 14.0 ; \\ 6.0 &\leq A \leq 9.0 ; \\ 2.2 &\leq H \leq 2.7 \end{aligned} \quad (4)$$

Here, values min-max of variable L correspond to the length range of small houses built in Mexico, as well as the width A . The adopted values of height is accorded to the anthropometric studies mentioned before related with the main height of the latin american man, and taking into account the maximum value adopted in construction of small houses in Mexico.

According to previous studies of anthropometry and ergonomics, taking into account the space required by the inhabitants of the house, we get as a minimum housing area of 65.0 m². In particular, we have considered the proxemics that involves the required distances between inhabitants and furniture inside house. Here the analysis was done considering 4 inhabitants in the house [12], when we add together all these distances we obtain the area value indicated before of 65.0 m².

On the other hand, the minimum volume air required by the inhabitants of housing has been established as mentioned above, at least 25.0 m³ each. So, these two conditions or state equation can be written as follow:

$$g_1(x) \geq 65.0 / (A \cdot L); \quad g_2(x) \geq 100 / (A \cdot L \cdot H) \quad (5)$$

The gain or loss of energy has been obtained on the basis of energy efficiency standards procedures, as well as changes in the walls and roof construction as mentioned above involving the thermal properties of each component. Final equation of gain energy has the range below shown. To arrive to this final equation we have considered and according to the Mexican Standards [13] a comparison between a *reference (benchmark) house* and the projected house, at the same time was considered all thermal effects on envelopes, i.e. walls, roof and vanes including windows and doors. Properties of conduction, convection and radiation have been included, film convection coefficients has taken according standards, finally our projected house shown below has been considered as mentioned an additional material type EPS to be in accordance with standard regulations. The procedure to arrive to this equation that can be seen on details on reference [12], have been included the conduction, convection and radiations effects on the house. This final equation relates the energy gain of our proposed house project versus benchmark house according Mexican Standards.

$$g_3(x) \geq \left[\left(1 - \frac{7.28 A \cdot L + 4A \cdot H}{10.83A \cdot L + 32.05 A \cdot H} \right) 100 \right] \quad (6)$$

$$g_4(x) \leq \left[\left(1 - \frac{7.28 A \cdot L + 4A \cdot H}{10.83A \cdot L + 32.05 A \cdot H} \right) 100 \right] \quad (7)$$

5. RESULTS

Geometric procedure was applied to obtain the minimum value of objective function F under the all mentioned above restrictions. In the Next

Fig. 2 can be seen some solutions. Equation (6) has two curved restricted lines. The red area is the area of feasible solutions. Any point inside it guarantee the required model conditions, but there is only one point where objective function F becomes lower (black lines), this point will be an optimal (local) minimum solution. On the Table 1 is indicated all main local minimum so obtained.

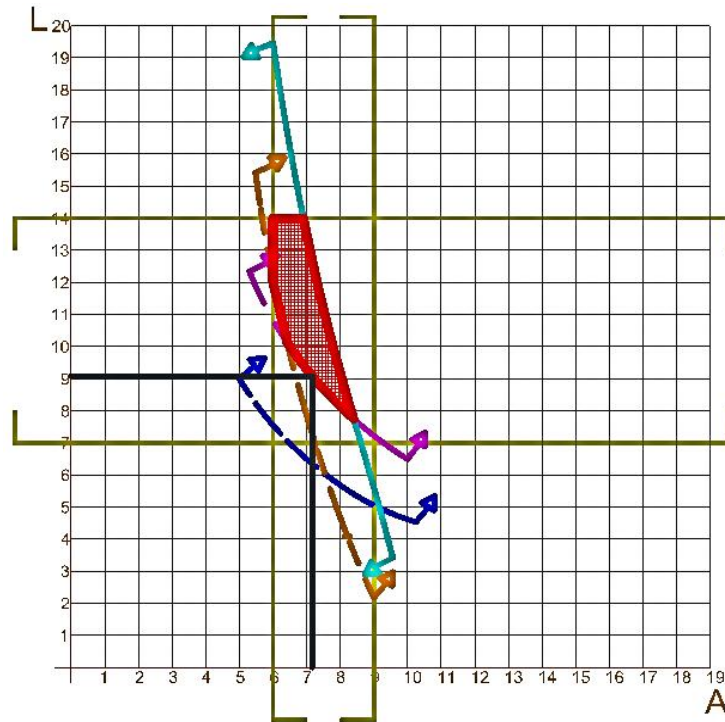
Table 1. List of optimal solutions

Optimal values of main dimensions of housing			
Width A (m)	Length L (m)	Height H (m)	Objective function F
7.17	9.0	2.2	141.97
6.68	9.72	2.3	149.34
6.06	10.67	2.4	155.18
6.5	10.0	2.5	162.5
6.08	10.67	2.6	168.67
6.7	9.67	2.7	174.93

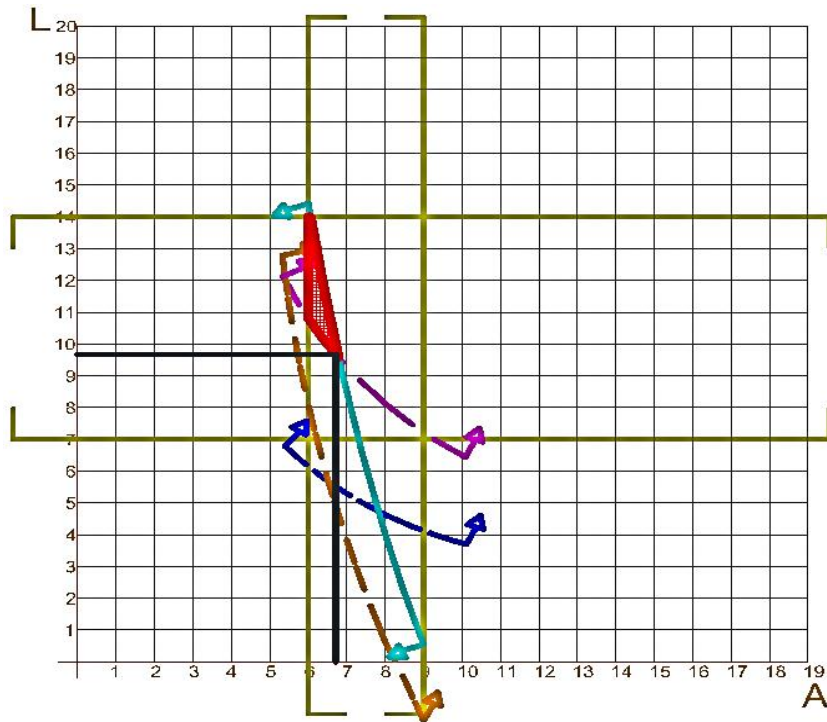
The behavior of the objective function varying the height H is indicated in Table 1, as well as the corresponding values of variables A and L , thereby obtaining the optimal dimensions of housing. Each triad (A - L - H) values shown in Table 1 correspond to the smallest value of the function F in the space of feasible solutions (red area). Any triad obtained is feasible and optimal, however in an optimization problem is preferably sought the global minimum of the objective function.

In this case it is observed that the value of objective function F decreases as the height H decrease, so the minimum H will correspond to the global minimum of the objective function. There are additional considerations that must be considered in the final choice of the proper height H . The ranges of the variables are important because they define the final behavior of the objective function. Here a lower value of height H may be appropriate for some people and circumstances, but cannot be generalized, since the height H of the house could be a significant factor of habitability for other people.

After defining the overall optimal dimensions of the house, we will now proceed to develop a proposal for distribution spaces inside housing, including equipment and furniture. In Fig. 3 is shown the distribution of housing adjusting the optimal dimensions thus obtained. In this case optimal area correspond to the lower value of objective function F associated to $H = 2.20$ m. As we have mentioned this height is suggested to be reviewed according to the specific conditions and circumstances of housing and its occupants.

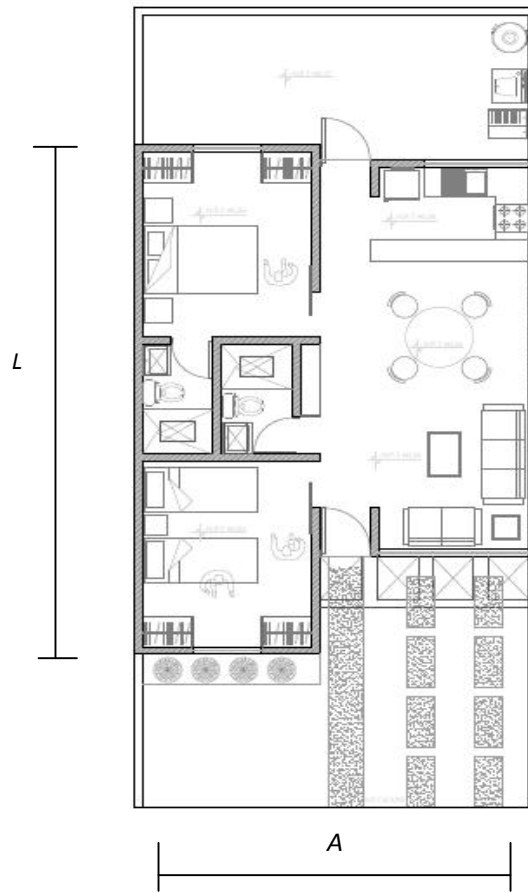


(a)



(b)

Fig. 2. Area of feasible solutions for varying L and A , maintaining H fixed. (a) Local minimum $H = 2.2$ m, $A = 7.17$ m, $L = 9.0$ m. (b) Local minimum $H = 2.7$ m, $A = 6.7$ m, $L = 9.67$ m



**Fig. 3. Housing final project according to obtained results. Optimal parameters:
 $A = 7.15$ m, $L = 9.0$ m. $H=2.2$ m**

6. CONCLUSIONS

The optimal dimensions obtained for the analyzed case depend on the height H , while decreasing the height decreases the objective function, however this relationship is not linear as can be seen on Figs. 2 (a) and 2 (b). Here the minimum of the objective function depends heavily on restrictions and State equations. The conditions under which this work was conducted correspond to a house in Mexico under the current Standards. Obtained results are interesting, but they are still unfinished. The case requires been solved by involving much more variables including the economic ones. Thus, the scope of the study carried out represent an improvement and a proposal for finding a more rational solution. Certainly, the construction of housing for low-income people have a tendency towards to decrease the size of the house and the inappropriate use of quality materials for reducing the cost of the house [12,14], another

alternative is to use the shown procedure for obtaining a more dignified living space with a minimum comfort for its inhabitants.

COMPETING INTERESTS

Authors have declared that no competing interests exist.

REFERENCES

1. Avalos L. Influence of internal habitability of housing in the quality of family life, Thesis. School of Psychology. UNAM. Mexico; 2003.
2. Hall TE. The Hidden Dimension, Doubleday Anchor Book, N.Y.; 1969.
3. Nguyen T, Reiter S. Optimum design of low-cost housing in developing countries using non smooth simulation-based optimization, 28th conference, Opportunities, Limits & Needs Towards an

- environmentally responsible architecture Lima, Peru; 2012.
4. Garth P. McCormick, Christoph Witzgall. Logarithmic SUMT limits in convex programming. Math. Program, Ser. A 90: 2001;113-145.
 5. National Institute of Statistics and Geography (INEGI); Available:www.inegi.org.mx
 6. Fonseca X. Anthropometry measures dwelling house, ED. Pax Mexico, Iztapalapa, DF; 2012.
 7. Monsalvo J. HV vital, habitability of housing and quality of life, Thesis. Fac. Of Psychology, UNAM. Mexico; 1998.
 8. Figue PL. Livability Housing social interest from the nineties; 2007.
 9. Neufert Ernest. Art of designing in architecture, 14^o Edición, Barcelona, Editorial Gustavo Gili, S. A.
 10. ONNCCE. "R" Value for housing envelopes thermal zone for the Mexican republic. Mexico: National Agency for Standardization and Certification of Building and Construction; SC; 2009.
 11. Brager G, Thermal adaptation in the built environment: a literatura review, Energy and buildings 27; 1998.
 12. Gomez Aime. Thesis Master Degree of Sciences. Construction. Engineering Department, Universidad Autonoma de Queretaro; 2014.
 13. NOM-008-ENER-2001. Energy Efficiency in Buildings. Non-residential buildings. Available:<http://www.energia.inf.cu/iee-mep/www/www.conae.gob.mx/nom008/index.htm>
 14. PE Maycotte, new types of social housing funded by government programs, memories XXVIII meeting of the national network of urban researchers; 2005.

© 2015 Rangel et al.; This is an Open Access article distributed under the terms of the Creative Commons Attribution License (<http://creativecommons.org/licenses/by/4.0>), which permits unrestricted use, distribution, and reproduction in any medium, provided the original work is properly cited.

Peer-review history:
The peer review history for this paper can be accessed here:
<http://sciencedomain.org/review-history/10342>