Vol. 12(2), pp. 24-30, July-December 2019
DOI: 10.5897/AJMCSR2019.0798
Article Number: 2AB7A7662382
ISSN: 2006-9731

http://www.academicjournals.org/AJMCSR

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African Journal of Mathematics and Computer Science Research

Full Length Research Paper

Total revenue function for non-regular fixed lifetime inventory system

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Received 11 September, 2019; Accepted 8 October, 2019

The fixed lifetime inventory system assumes that outdated items are discarded from inventory. Such inventory system is here referred to as regular inventory system. The non-regular fixed lifetime inventory system assumes that outdated items are not discarded from inventory. The features of both the regular and non-regular inventory systems are compared in this work and the total revenue function for the non-regular fixed lifetime inventory system is derived.

Key words: Regular, inventory, revenue, outdate, discarded.

INTRODUCTION

The utility of items under the fixed lifetime inventory system is constant. Their useful lifetime is known and fixed. Any of the item(s) not used to meet demand at the end of its useful lifetime, outdates and must be discarded from inventory at a loss to the inventory manager. Inventory system where items are outdated and are discarded from the system are classified as regular fixed lifetime inventory system. Authors in the literature that have addressed regular fixed lifetime inventory system include Nahmias and Pierskalla (1973), Cohen (1975), Fries (1975), Nahmias (1982), Chiu (1995), Liu and Lian (1999), Goyal and Giri (2000), Omosigho (2002), Mohammad et al. (2007), Olsson and Tydesjo (2010), Nahmias (2011), Tripathi and Shweta (2013), Aris (2014), Harshal et al. (2015), Izevbizua and Omosigho (2015), Shen-chih et al. (2016), Izevbizua and Omosigho (2017), Izevbizua and Apanapudor (2019), and Izevbizua and Emunefe (2019). All of these authors made the assumption that items that are outdated are discarded from the inventory. Examples of regular fixed lifetime inventory systems are blood inventory, egg inventory, pharmaceutical inventories, foodstuff inventories, chemical inventories, etc.

The non-regular fixed lifetime inventory system is one in which outdated item(s) are not discarded from the inventory system. This is because item(s) that are outdated in a period can become useful in the next period. Items in inventory are only depleted by demand, unlike the regular fixed lifetime inventory system where items are depleted by both demand and expiration. The maximum number of items Q available to meet demand is constant for any non-regular fixed lifetime system and can be subdivided into units. Examples of non-regular fixed lifetime inventory system include hotel

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accommodation, where the total number of rooms available to meet demand is subdivided into suits, standard, double and single rooms. Seats on the airplane, where the total number of seats is available is subdivided into first class, business class and economic class. Advert space on the daily newspapers, which can be classified as full page, half page, quarter page, etc. Television advert/programme where advert/programme can run for 1 h, ½ h, 3/4 h … 1 min. The price for an item differs from subunit to subunit and the total revenue generated is the sum of revenue from all units.

A review of the fixed lifetime literature by Izevbizua and Omosigho (2017) shows that the regular fixed lifetime inventory system has been extensively studied, while the non-regular fixed lifetime inventory system has received little or no attention from the research community. The aim of this work is to examine the non-regular fixed lifetime inventory system and derive the equation for the total revenue generated by the non-regular fixed lifetime inventory system.

Features of regular fixed lifetime inventory system

(1) Items have a defined number of useful periods, known as the lifetime of the product. Any item not used to meet demand at the end of this defined useful period is outdated and must be discarded.
(2) The utility of items is constant throughout their useful periods in inventory.
(3) Examples of regular fixed lifetime products include eggs, bread, blood, pharmaceuticals, chemicals, etc.
(4) Inventory is depleted by demand and outdating.
(5) The quantity of items available to meet demand is not fixed, it depends on the order size.
(6) Items available to meet demand are all of the same type and are not subdivided into units.

Features of non-regular fixed lifetime inventory system

(1) Items do not have a defined useful lifetime. Items are assumed to outdate if not used to meet demand at the end of any period. However, the utility of the item is restored at the start of the next period.
(2) The items have piecewise utility.
(3) Examples of non-regular fixed lifetime products are hotel rooms, space on daily newspapers, seats on airplane, space on buses or trains, airtime on television and radio.
(4) Inventory is only depleted by demand.
(5) The quantity of items available to meet demand is fixed. The number of rooms in a hotel and the seats on an airplane are fixed.
(6) The items available to meet demand are subdivided into units.

DESCRIPTION OF MODEL

The total number of items Q available to meet demand in a non-regular fixed lifetime inventory is fixed. The number of rooms in the hotel, seats on the airplane, spaces on the daily newspaper and number of hours a television/radio station runs daily are fixed. Also, the items are divided into subunits $q_1, q_2, q_3, \ldots$ and the number of items used to meet demand in each subunit is $a_1, a_2, a_3, \ldots$ Table 1 shows revenue obtainable for some non-regular fixed lifetime inventory system.

Table 1 shows the total number of items Q available to meet demand and the subunits q_i of Q for some non-regular fixed lifetime inventory. Also, the number of items used to meet demand from each subunit in a period is given by a_i. a_i can be all of q_i for a particular subunit or part of q_i. For example, if there are 20 suits in a hotel, then a_1 for any period will take value from the interval [0,20]. 0 if no suit is used to meet demand and 20 if all the suits are used to meet demand. $\bar{a}_i$ is the number of items used from q_i to meet demand and will remain in demand for k periods.

Notation

Q= maximum number of items available to meet demand.

$q_i, i = 1,2,3,\ldots n$ represents the subunits of Q. With $q_1 + q_2 + \ldots + q_n = Q$

$q_i = a_i + \bar{a}_i$ where $\bar{a}_i$ are items in demand and a_i are items available to meet demand.

$x_i, i = 1,2,3,\ldots n$ represents the unit price of items in the subunits.

$a_i, i = 1,2,3,\ldots n$ =number of items available to meet demand from a subunit.

$\bar{a}_i, i = 1,2,3,\ldots n$ =number of items in demand from a subunit.

n =number of subunits

t_i = demand in period i.

b_i = item(s) used to meet demand in period i and is available to meet demand in period $i+1$

p_i = period i

k = number of periods any item(s) is not available to meet demand.

Table 1. Revenue obtainable from some non regular fixed lifetime inventory system.

S/N	Available items Q	Subunit q_i	Number of items from q_i	Price per unit x_i	Revenue from subunits	Total revenue
1	Hotel Rooms	Suits (q_1)	a_1	x_1	a_1x_1	$\sum_{i=1}^{4} a_i x_i$
		Standard (q_2)	a_2	x_2	a_2x_2	
		Double (q_3)	a_3	x_3	a_3x_3	
		Single (q_4)	a_4	x_4	a_4x_4	
2	Seats on airplane	First class (q_1)	a_1	x_1	a_1x_1	$\sum_{i=1}^{3} a_i x_i$
		Business class (q_2)	a_2	x_2	a_2x_2	
		Economic class (q_3)	a_3	x_3	a_3x_3	
3	Spaces on daily newspaper	Full page (q_1)	a_1	x_1	a_1x_1	$\sum_{i=1}^{5} a_i x_i$
		Half page (q_2)	a_2	x_2	a_2x_2	
		¼ page (q_3)	a_3	x_3	a_3x_3	
		1/8 page (q_4)	a_4	x_4	a_4x_4	
		1/16 page (q_5)	a_5	x_5	a_5x_5	
4	Hours on television/radio	2 h (q_1)	a_1	x_1	a_1x_1	$\sum_{i=1}^{6} a_i x_i$
		1 h (q_2)	a_2	x_2	a_2x_2	
		30 min (q_3)	a_3	x_3	a_3x_3	
		20 min (q_4)	a_4	x_4	a_4x_4	
		10 min (q_5)	a_5	x_5	a_5x_5	
		1 min (q_6)	a_6	x_6	a_6x_6	

p = set up cost.
λ = shortage cost.
θ = outdate cost.
$f(t)$ = demand distribution.

Table 2 shows the quantity of items available to meet demand at the start of a new period. The number of items available in the first three periods of the system is given and the expression for period n.

The total number of units available to meet demand is Q. The demand at the end of the first period is t_1 and b_1 represent items used to meet demand in period 1 and available to meet demand in period 2. The number of items on hand at the start of period 2 is

$Q - t_1$
or
$Q + b_1 - t_1$

Again t_2 is the demand in period 2 and b_2 represents items used to meet demand in period 2 and are available to meet demand in period 3. The number of items on hand at the start of period 3 is:

Table 2. Quantity of items available to meet demand at the start of new periods.

p_1	p_2	p_3	p_4	. . .	p_n
Q	$Q-t_1$ $Q+b_1-t_1$	$(Q-t_1)^+-t_2$ $(Q+b_1-t_1)^++b_2-t_2$	$(Q-t_1-t_2)^+-t_3$ $(Q+b_1-t_1+b_2-t_2)^++b_3-t_3$	-	-

$$(Q-t_1)-t_2$$
$$or$$
$$(Q+b_1-t_1)+b_2-t_2$$

Continuing the process, the number of items on hand at the start of period n is

$$Q-t_1-t_2-...t_n$$
$$or$$
$$(Q+b_1+b_2+...+b_n)-t_1-t_2-...t_n$$

Example 1: The total number of rooms in a hotel is $Q=20$.

The demand at the end of period 1 is 15 rooms and 5 out of the 15 rooms will be available to meet demand in period 2. So the number of rooms available at the start of period 2 is

$$Q+b_1-t_1=20+5-15=10$$

If the demand in period 2 is 4 rooms and 2 out of the four will be available to meet demand in period 3, then number of rooms available at the start of period 3 is

$$(Q+b_1-t_1)+b_2-t_2=10+2-4=8.$$

The process continues and the number of rooms available to meet demand at the start of every period can be determined.

Assumptions of the model

(1) Q (fixed) is the maximum number of items available to meet demand and Q can be subdivided into subunits.
(2) Satisfied demand in a period cannot exceed Q.
(3) Some items used to meet demand in period i, can become available to meet demand in period $i+1$. While some items used to meet demand in period i may not be available for some defined period k.
(4) Anytime demand is greater Q, the excess demand is lost at a cost against the inventory manager.
(5) Any item(s) not used to meet demand at the end of a period is considered outdated.
(6) Total revenue is the sum of revenues from all subunits.
(7) There is a constant setup cost.
(8) The outdate cost and shortage cost are the same for all subunits.

Derivation of total revenue function

The total revenue function for the system is derived. The component of the revenue function is the set up cost, revenue generating function, shortage cost and outdate cost.

Setup cost

There is a constant set up cost P for the model.

Revenue generating function

Revenue is generated through the sales of items in inventory. The total available item Q is subdivided into units and each unit has its unit price. Items from each subunit are further divided into items in demand and items available to meet demand.

$$q_1=a_1+\overline{a_1}$$
$$q_2=a_2+\overline{a_2}$$
$$.$$
$$.$$
$$.$$
$$q_n=a_n+\overline{a_n}$$

This implies that

$$Q=q_1+q_2+...+q_n=a_1+\overline{a_1}+a_2+\overline{a_2}+...+a_n+\overline{a_n}$$
$$Q=a_1+a_2+...+a_n+\overline{a_1}+\overline{a_2}+\overline{a_n}$$
$$Q=\sum_{i=1}^{n}(a_i+\overline{a_i})$$

Introducing the unit price per subunit and the number of period k an item is in demand, gives

$$\begin{aligned} \mathrm{Re}venue &= a_1x_1 + a_2x_2 + \ldots + a_nx_n + k_1\overline{a}_1x_1 + k_2\overline{a}_2x_2 + \ldots + k_n\overline{a}_nx_n \\ &= \sum_{i=1}^{n} a_ix_i + \sum_{i=1}^{n} k_i\overline{a}_ix_i \\ &= \sum_{i=1}^{n}\left(a_ix_i + k_i\overline{a}_ix_i\right) \end{aligned} \quad (1)$$

The present investigation reveals that, sometimes no item from a given subunit is in demand (that is all items from the subunits are available to meet demand). For example, if no items from q_1 *and* q_2 are in demand, then Equation 1 becomes

$$\mathrm{Re}venue = \sum_{i=1}^{n} a_ix_i + \sum_{i=3}^{n} k_i\overline{a}_ix_i \quad (2)$$

Also, if all the items that make up Q are in demand (that is, no items is available to meet demand), then,

$$\mathrm{Re}venue = \sum_{i=1}^{n} k_i\overline{a}_ix_i \quad (3)$$

Shortage cost

Shortage occurs whenever demand in a period t exceeds items available to meet demand in the period. By the assumption of the model, all excess demand is lost at a cost to the inventory manager. Hence, shortage occurs when $t > Q$ and shortage quantity per period is given by:

$$Shortage = \int_{Q}^{\infty} (t-Q)\, f(t)\, dt \quad (4)$$

With a shortage cost of λ per unit, shortage cost is

$$Shortage\ \ \mathrm{cost} = \lambda\int_{Q}^{\infty} (t-Q)\, f(t)\, dt \quad (5)$$

Outdate cost

Items are outdated if they are not used to meet demand at the end of a period. Outdating implies that demand in a period is less than the items available to meet demand in that period. Hence, outdating occurs if $Q > t$ and the outdate quantity per period is given as

$$Outdate = \int_{0}^{Q} (Q-t)\, f(t)\, dt \quad (6)$$

With an outdate cost of θ per unit, outdate cost is

$$Outdate\ \mathrm{cost} = \theta\int_{0}^{Q} (Q-t)\, f(t)\, dt \quad (7)$$

Therefore our total revenue function (TRF) per period is

$$TRF = P + \sum_{i=1}^{n}(a_ix_i + k_i\overline{a}_ix_i) - \lambda\int_{Q}^{\infty}(t-Q)\, f(t)\, dt - \theta\int_{0}^{Q}(Q-t)\, f(t)\, dt \quad (8)$$

where $f(t)$ is the Poisson distribution of the demand.

The solution of the model was obtained via a computer programme written in MATHEMATICA. Input the parameter values to generate solutions.

Numerical example

The model was applied by a hotel in Benin City, Nigeria. The result is shown in Table 3. Table 3 gives the total number of rooms available in the hotel and the number of rooms available in each subunit. Also, the unit cost for rooms in each subunit is shown in Table 3.

Table 4 shows the total revenue generated by the hotel in four days (4, 5, 6 and 7th) in April 2018. Day 1 (4th April, 2018); the number of suits available to meet demand was 15 while 5 (2 for two days and 3 for four days) were in demand. The demand for day 1 was 10 (2 for three days, 1 for four days and 7 for 1 day).

The number of standard available to meet demand was 18 while 2 were in demand for one day each. The demand for the day was 8, each for one day.

The number of double available to meet demand was 28 while 7 were in demand for a day each. The demand for the day was 10 (2 for 3 days each, 2 for 4 days each and 6 for 1 day each).

Finally, for day 1, the number of single available to meet demand was12 while 13 were in demand. The demand for the day was 6, each for a day. Continuing, days 2, 3 and 4 can be similarly analyzed.

DISCUSSION

The total revenue function for the non-regular fixed lifetime inventory can be used to determine the daily

Table 3. The total number of rooms in the hotel and cost prices (Naira).

$Q=100$	Suits=20 Standard=20 Double=35 Single=25
Price per unit per day	Suits=20,000 Standard=10,000 Double=8000 Single=4000
Outdate cost per unit	1000 (for all units)
Shortage cost per unit	1000 (for all units)

Table 4. Revenue generated by the hotel in four days.

Day	Number of items per subunit	Items available to meet demand	Items in demand	Daily demand	Revenue from subunit (Naira)	Outdates	Shortages	Total revenue per day (Naira)
1	Suits=20	15	$5\|^{2(2)}_{3(4)}$	$10\|^{2(3)}_{1(4)}$	340000	5	-	565000
	Standard=20	18	2	8	80000	10	-	
	Double=35	28	7	$10\|^{2(3)}_{2(4)}$	160000	18	-	
	Single=25	12	13	6	24000	6	-	
2	Suits=20	12	$8\|^{1(2)}_{4(3)}$	13	260000	-	1	434000
	Standard=20	20	-	10	100000	10	-	
	Double=35	31	$4\|^{1(2)}_{1(3)}$	15	120000	12	-	
	Single=25	25	-	$12\|^{2(3)}_{4(2)}$	80000	13	-	
3	Suits=20	15	$5\|^{1(1)}_{4(2)}$	$15\|^{2(4)}_{3(2)}$	480000	-	-	773000
	Standard=20	20	-	$12\|^{2(5)}$	200000	8	-	
	Double=35	31	$2\|^{1(1)}_{1(2)}$	10	80000	21	-	
	Single=25	19	$6\|^{2(2)}_{4(1)}$	12	48000	7	-	
4	Suits=20	11	$9\|^{7(1)}_{2(3)}$	$8\|^{1(3)}_{2(2),3(4)}$	420000	3	-	855000
	Standard=20	18	$2\|^{(4)}$	15	150000	3	-	
	Double=35	34	1	24	192000	10	-	
	Single=25	23	2	28	112000	-	3	

revenue generated by any non-regular inventory system. The revenue generated by each subunit is known and the decision to increase or decrease the capacity of a subunit can be based on the revenue generated by the subunit.

The profitability of the system can determined from the revenue generated over a period of time.

Conclusion

The total revenue function for non-regular fixed lifetime inventory system was derived. The function enables the inventory manager to compute the amount of money generated by the subunits every period. It also helps determine the number of items not used to meet demand at the end of each period and the shortages resulting from excess demand. Also, the model was implemented on a hotel, which is classical non-regular fixed lifetime inventory system.

CONFLICT OF INTERESTS

The authors have not declared any conflict of interests.

REFERENCES

Aris AS (2014). Perishable inventory systems. Journal of the Operational Research Society 65:797-798.

Chiu HN (1995). An Approximation to the Continuous review Inventory Model with Perishable Items and Lead Times. European Journal to Operational Research 87(1):93-108.

Cohen MA (1975). Analysis of Single Critical Number Ordering Policies for Perishable Inventories. Operations Research 24(4):726-741.

Fries BE (1975). Optimal Ordering policy for a Perishable Commodity with fixed Lifetime. Operations Research 23(1):46-61.

Goyal SK, Giri BC (2000). Recent trends in Modeling of Deteriorating Inventory. European Journal of Operational Research 134 (1):1-16.

Harshal L, Rahul N, Ravichandran N (2015). Analysis of an order-up-to-level policy for perishable with random issuing. Journal of the operations research society 67(3):483-505.

Izevbizua O, Apanapudor J (2019). Comparing the sensitivity of quantity based and useful lifetime based fixed lifetime inventory models to changes in cost. The Journal of the Mathematical Association of Nigeria 46(1):323-331.

Izevbizua O, Emunefe OJ (2019). Statistical analysis of bread inventory, using the useful lifetime based ordering policy. Transactions of the Nigeria Association of Mathematical Physics 9:81-84.

Izevbizua O, Omosigho SE (2015). The convexity of the total cost function for a fixed lifetime inventory system. The Transactions of the Nigeria Association of Mathematical Physics 1:227-236.

Izevbizua O, Omosigho SE (2017). A review of the fixed lifetime inventory system. The Journal of the Mathematical Association of Nigeria 2:188-198.

Liu L, Lian Z (1999). (s,S) Continuous Review Models for Products with Fixed Lifetime. Operations Research 47(1):150-158.

Mohammad AR, Manuel DR (2007). An efficient heuristic optimization algorithm for a two-echelon inventory system. International Journal of Production Economics 109(1-2):195-213.

Nahmias S (1982). Perishable Inventory Theory: A Review. Journal of Operations Research 30(4):680-708.

Nahmias S (2011). Perishable inventory systems. International series in operations research and management science Vol. 160.

Nahmias S, Pierskalla WP (1973). Optimal Ordering Policies for a Product that Perishes in two Periods Subject to Stochastic Demand. Naval Research Logistics Quarterly 20(2):207-229.

Olsson F, Tydesjo P (2010). Inventory Problems with Perishable items: Fixed Lifetimes and Backlogging. European Journal of Operational Research 202(1):131-137.

Omosigho SE (2002). Determination of Outdate and Shortage Quantities in the Inventory Problem with fixed Lifetime. International journal of computer mathematics 79(11):1169-1177.

Shen- Chih C, Jie M, Jinn-Tsair T, Fuan Li (2016). Inventory and Shelf - Space optimization for fresh produce with expiration date under freshness- and -stock- dependent demand rate. Journal of the Operational Research Society 67(6):884-896.

Tripathi RP, Shweta ST (2013). Optimal order policy for time-dependent Deteriorating items in response to temporary price discount linked to order quantity. Journal of applied mathematical sciences 7:2869-2878.