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The Four Aspects of Matter and Radiation

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The sole author designed, analyzed and interpreted and prepared the manuscript.

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ABSTRACT

The idea is advocated here that there are four possible 'aspects' for matter to manifest in whatever reference frame because Special Relativity admits four variations to standard Lorentz transformations: two basic variations, bradyonic and pseudotachyonic transformations, applying to respectively subluminal and superluminal reference frames; and two others, derived from these ones by simply reversing time. Pseudotachyonic Relativity (PtR), proposed eight years ago, show that even though one cannot directly detect particles moving faster-than-light, one can detect their co-particles, their 'images' moving slower-than-light but with opposite energy, mass and charge; in the process, negative energies naturally arise in Special Relativity, which is quite relevant in field theory. One also concludes that time flows in two opposite senses in the Universe and this is why classic theories are essentially time-reversible. The news here come from the discussion of Dirac equation for the electron and how negative energy turns into positive; one discovers that this equation applies as well to negative mass and finally that its positive and negative solutions are related by 'antibradyonic' Lorentz transformation. Generally, in terms of Relativity, this explains why each particle has its own antiparticle. And not just that. In fact, Dirac equation agrees with the proposed Quadrivalent Special Relativity in the conclusion that each particle, in a wide sense, may appear (or manifest itself) in one of four aspects, four versions of a single root – its

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'archeparticle' –, depending on its mass-energy signature: 'straight' particle; antiparticle (with negative-energy): PtR co-particle (also with negative-energy); and co-antiparticle. This conclusion also applies to massless particles, such as photons, with an equivalent alignment-energy signature.

Keywords: Special relativity; pseudotachyonic relativity; negative energy; reversing time; De Broglie wave; Dirac equation; massless particles.

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1 INTRODUCTION

In three former articles, [1], [2], [3], I proposed the theory of Pseudotachyonic Relativity (PtR) consisting on an extension of Lorentz transformations to $|v| > c$. According to this theory, we'll review in a while, tachyonic reference frames actually exist but they can only be detected as pseudotachyonic ones, moving with lower-than-light velocity $\hat{v} = c^2/v$.

In what comes to Lorentz transformations, a pseudotachyonic frame behaves 'almost' like a bradyonic one but in it time appears reverted. This 'tiny' difference is crucial. In fact, this conclusion apply as well to a tachyonic particle [or *tachyon*, a putative particle moving faster-than-light [4]]; but then, this particle appears as its pseudotachyonic 'image', with negative energy and inverse electric charge. That's why I associated it with antimatter. Furthermore, this point of view provided a very satisfying explanation for the fundamental question: why does every particle have an homologous antiparticle? Well, simply because the antiparticle is nothing more than the detection of a tachyonic homologous particle; there is no difference in nature between a particle and its homologous one, the apparent difference is nothing but a relativistic effect owing to its state of movement relatively to the reference frame that evaluates it.

The theory appears to be consistent, even though it presents some controversial conclusions with regard to standard Physics. Obviously, I soon realised that this conception of antimatter differs from the standard one, originally presented by Paul Dirac. In fact, we draw from PtR theory some quite different conclusions, for instance concerning negative masses and consequent repulsive gravity or the threshold energy in the pair creation process, which simply doesn't exist for PtR 'antiparticles'.

Sure, the (apparent) contradictions between PtR and canonical theories are significant. This is quite troublesome but should one discourage or simply discard PtR? I don't think so because the theory consistently expands Special Relativity and opens the door to some remarkable conclusions, abolishing interdicts and asymmetries (for instance, proposing negative absolute temperatures to PtR 'antimatter'), in fact expanding frontiers for Physics research and the comprehension of the Universe. That's why I never disbelieved or abandoned it, that's why, for a long time, now and then I struggled with internal issues and external contradictions, in what comes to Quantum theory but also to General Relativity. I think I finally discovered the basic answers and how to advantageously overcome these dilemmas and move forward. I will not speak here about GR but mainly about Dirac theory of the electron, which I reviewed and which, in fact, brings at least these surprising outcome: 1) PtR and Dirac theory, even though having the same root (i.e., Special Relativity), present conceptions of antimatter which concern related but different 'things'; 2) Dirac equation is even richer than we thought because it also applies to PtR 'anti-electron'. In summary, there isn't any conflict between both theories – they simply become larger!

The primary goal of this reflection is precisely to understand how and why there are two consistent types of antimatter. We'll analyse this issue beginning with a review of PtR theory basics, passing by de Broglie waves and his "periodic phenomenon", which proves to be fundamental in the creation of force fields. A critical reflection on Dirac equation follows. We'll finish with the proposal of Antibradyonic Relativity, which brings a major coherence to all the picture and allows us to understand that behind the multiplicity of homologous particles there is a single entity, which we may call their “archeparticle”. And this is most gratifying!

A final note: some time ago I discovered that PtR ‘antimatter’ largely meets the conception of the creator of the word “antimatter”: *“The term antimatter was first used by Arthur Schuster in two rather whimsical letters to Nature in 1898, in which he coined the term. He hypothesized antiatoms, as well as whole antimatter solar systems, and discussed the possibility of matter and antimatter annihilating each other. Schuster’s ideas were not a serious theoretical proposal, merely speculation, and like the previous ideas, differed from the modern concept of antimatter in that it possessed negative gravity”*[5]. Even if antimatter in the PtR conception really earned its name, because it appears opposite to matter in all characteristics, I know it’s useless to try to ‘dethrone’ Dirac antimatter – which should be called otherwise. Instead, I will name “co-matter” the former PtR ‘antimatter’.

2 PSEUDOTACHYONIC RELATIVITY

2.1 Lorentz Transformations

The theory of Pseudotachyonic Special Relativity begins with an extension of Lorentz standard transformations to $|v| > c$. As I vindicated in [1], there is no scientific reason to exclude the possibility for tachyonic frames to exist. The fundaments of Special Relativity – the principle of relativity and the postulate of the constancy of the speed of light – were established by Einstein himself without any restriction concerning the relative velocity of reference frames [6]. I proposed then to extend these principles to *all* inertial reference frames, considering all of them equivalent: *“The principle of equivalence immediately means that we must not conceive a tachyonic frame as something quite peculiar, where strange things happen. On the contrary, it is as trivial as any bradyonic frame. Coordinates, for instance, physically mean exactly what they mean in our own reference frame or in any bradyonic one. It’s very important to understand that there is no fundamental difference in nature between bradyonic and tachyonic reference frames. Bizarre results appear only when we try to relate both frames”* [1].

We may deduce the standard Lorentz transformations through the fundamental invariance of the Minkowski line element ds, given [with metric signature $(-+++)$] by the quadratic equation $ds^2 = (ic\,dt)^2 + dx^2 + dy^2 + dz^2$. Remark that this is mathematically valid for whatever value of $|v| \neq c$, since neither the deduction of the transformations, according to the principles of Relativity, nor the invariance of the line element ds depend on the restriction $|\beta| < 1$. However, from a physical point of view, for $|\beta| > 1$ this assumption concerns a reinterpretation of Lorentz transformations related to the fact that one cannot directly detect a tachyonic frame.

Let S be ‘our’ bradyonic frame of coordinates and S'' a tachyonic frame moving in S along the xx axis with velocity v, $|v| > c$; the standard Lorentz transformations give

$$\begin{cases} t'' = \frac{t - x\beta/c}{i\alpha} \\ x'' = \frac{x - \beta ct}{i\alpha} \\ y'' = y \\ z'' = z \end{cases} \quad \text{making } \alpha = \sqrt{\beta^2 - 1}. \quad (2.1)$$

In order to eliminate the imaginary numbers which arise for the measurable variables of space and time in this ordinary transformation (and which, in fact, physically mean the impossibility of directly detect a tachyonic reference frame), I think we have no alternative but adopt the following idea: a tachyonic frame S'' may only be detected as its *associated frame*, said *pseudotachyonic* and symbolised by S^*, obtained from the first by an interchange of space-time axis

$$\begin{cases} x^* = -ict'' \\ ict^* = -x'' \end{cases} \quad \text{for } \beta > 1 \quad (2.2)$$

and which proves (see below) to have lower-than-light velocity:

$$\hat{v} = \frac{c^2}{v}\,.$$

Applying (2) in (1), we obtain the Lorentz pseudotachyonic transformations for $\beta = \frac{v}{c} > 1$, relating the pseudotachyonic frame S^* to S:

$$\begin{cases} t^* = \frac{x/c - \beta t}{\alpha} \\ x^* = \frac{x\beta - ct}{\alpha} \\ y^* = y \\ z^* = z \end{cases} \quad \text{in which } \alpha = \sqrt{\beta^2 - 1} \quad (\text{for } \beta > 1)\,. \tag{2.3}$$

Generally speaking, and adopting tensorial notation, we'll consider the contravariant space-time 4-vector in the form $\mathbf{x}^\mu = (ict,\, x,\, y,\, z,)$ instead of the usual $\mathbf{x}^\mu = (ct,\, x,\, y,\, z,)$; this is because the relevant time-coordinate in the invariant ds^2 is ict and not ct. In fact, it is precisely the presence of the imaginary unit i in the 'time' axis that gives it a distinctive status. We'll also generalize this to any generic contravariant 4-vector $\mathbf{A}^\mu$, considering implicitly that the coordinate $\mathbf{A}^0$ is a pure imaginary number. For $|v| < c$, this generic 4-vector transforms according to the bradyonic rule [7]:

$$\begin{cases} \mathbf{A}'^0 = \frac{\mathbf{A}^0 - i\beta\mathbf{A}^1}{\sqrt{1-\beta^2}} \\ \mathbf{A}'^1 = \frac{\mathbf{A}^1 + i\beta\mathbf{A}^0}{\sqrt{1-\beta^2}} \\ \mathbf{A}'^2 = \mathbf{A}^2 \\ \mathbf{A}'^3 = \mathbf{A}^3. \end{cases} \tag{2.4}$$

One must be aware of the i factor, not only in the timelike-coordinate $\mathbf{A}^0$ but also in the first two equations above. Because of its presence, we are allowed to accept that these transformations also apply to any S'' tachyonic. But then, if S^* is its associated frame, we'll establish the symmetrical conditions:

$$\begin{cases} \mathbf{A}^{*0} = -\mathbf{A}''^1 \\ \mathbf{A}^{*1} = -\mathbf{A}''^0 \end{cases} \quad \text{for } \beta > 1 \quad \text{and} \quad \begin{cases} \mathbf{A}^{*0} = \mathbf{A}''^1 \\ \mathbf{A}^{*1} = \mathbf{A}''^0 \end{cases} \quad \text{for } \beta < -1. \tag{2.5}$$

From now on we'll focus on $\beta > 1$ (for $\beta < -1$ and the signals option, consult the Appendix A in [1]); the consequence is that the standard *pseudotahyonic transformation rule* for the 4-vector $\mathbf{A}^\mu$ must be writen as:

$$\begin{cases} \mathbf{A}^{*0} = \frac{i\mathbf{A}^1 - \beta\mathbf{A}^0}{\alpha} \\ \mathbf{A}^{*1} = \frac{i\mathbf{A}^0 + \beta\mathbf{A}^1}{\alpha} \\ \mathbf{A}^{*2} = \mathbf{A}^2 \\ \mathbf{A}^{*3} = \mathbf{A}^3, \end{cases} \quad \text{for } \beta > 1. \tag{2.6}$$

Note that the inverse transformation is similar to this one (in fact, the similitude between equations for inverse transformations is a characteristic of Pseudotachyonic Relativity, in connexion with the transformation of velocities):

$$\begin{cases} \mathbf{A}^0 = \frac{i\mathbf{A}^{*1} - \beta\mathbf{A}^{*0}}{\alpha} \\ \mathbf{A}^1 = \frac{i\mathbf{A}^{*0} + \beta\mathbf{A}^{*1}}{\alpha} \\ \mathbf{A}^2 = \mathbf{A}^{*2} \\ \mathbf{A}^3 = \mathbf{A}^{*3}, \end{cases} \quad \text{for } \beta > 1. \tag{2.7}$$

As it is known, we can express the four equations in (2.5) – in fact, any Lorentz transformations – simply as a matrix product:

$$\mathbf{A}' = \mathbf{\Lambda}'\mathbf{A}, \tag{2.8}$$

where $\mathbf{\Lambda}'$ is the Lorentz matrix adapted to the imaginary components $\mathbf{A}^0$, with $\gamma = \frac{1}{\sqrt{1-\beta^2}}$:

$$\mathbf{\Lambda}' = \begin{pmatrix} \gamma & -i\gamma\beta & 0 & 0 \\ i\gamma\beta & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}. \tag{2.9}$$

Following the conditions (2.6), for $\beta > 1$ the pseudotachyonic transformations

$$\mathbf{A}^* = \mathbf{\Lambda}^* \mathbf{A} \tag{2.10}$$

simply results from changing signs in the first two lines of $\mathbf{\Lambda}'$ and permuting them:

$$\mathbf{\Lambda}^* = \begin{pmatrix} -i\gamma\beta & -\gamma & 0 & 0 \\ -\gamma & i\gamma\beta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} -\frac{\beta}{\alpha} & \frac{i}{\alpha} & 0 & 0 \\ \frac{i}{\alpha} & \frac{\beta}{\alpha} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}, \tag{2.11}$$

since $\gamma = -\frac{i}{\alpha}$.

Now, if we apply the first condition of (2.5) – corresponding to (2.2) – to the transformation of the space-time 4-vector, we'll obtain the four equations (2.3); and from these ones we derive the equations for the composition of velocities:

$$\begin{cases} u_x^* = \frac{u_x\beta - c}{u_x - v} c \\ u_y^* = \frac{u_y\alpha}{u_x - v} c \\ u_z^* = \frac{u_z\alpha}{u_x - v} c. \end{cases} \tag{2.12}$$

Of course, the constancy of the speed of light is preserved in pseudotachyonic transformations – as one should expect, since it is a basic foundation of Special Relativity. However, it results the singular reversion:

$$\begin{cases} u = c & \Leftrightarrow & u^* = -c \\ u = -c & \Leftrightarrow & u^* = c. \end{cases} \tag{2.13}$$

In general, if the velocity $\mathbf{u}$ is parallel to the xx axis, we may write

$$u^* = \frac{c^2 - uv}{v - u} \quad \text{and, identically,} \quad u = \frac{c^2 - u^*v}{v - u^*}; \tag{2.14}$$

or write both equations in a single one,

$$uv + u^*v - uu^* - c^2 = 0, \tag{2.15}$$

which is (as in bradyonic transformations) the equation of an equilateral hyperbole [1]. In particular, we conclude that

$$u^* = 0 \quad \Leftrightarrow \quad u = \frac{c^2}{v},$$

and this means that the tachyonic frame S'' must be detected in S as its *associated frame* S^*, which has a subluminal velocity $\hat{v}$, called *associated velocity* or *co-velocity* of v (read *"hat v"* or *"co-v"*), given by:

$$\hat{v} = \frac{c^2}{v}; \tag{2.16}$$

this is,

$$\widehat{\beta} = 1/\beta \quad \text{or} \quad v\hat{v} = c^2. \tag{2.17}$$

As vectors, $\hat{\mathbf{v}}$ is parallel to $\mathbf{v}$ and has a magnitude $\hat{v} = c^2/v$; therefore, $\hat{\mathbf{v}} = \widehat{\beta}^{\,2}\mathbf{v}$ and $\hat{\mathbf{v}} \cdot \mathbf{v} = c^2$.

Remark that, identically,

$$u = 0 \quad \Leftrightarrow \quad u^* = \frac{c^2}{v};$$

the conclusion is that the frame S is detected in S^* moving with the same velocity $\hat{v}$ – and not its inverse, as one should expect –, and this is the reason why inverse pseudotachyonic transformations result identical. In fact, the phenomenon of time reversion implicated in the transformation $S \leftrightarrow S^*$, as we'll see below, is the fundamental cause of an identical velocity $\hat{v} = c^2/v$ for the mutual detection of S and S^*.

Consider now a bradyonic frame S' moving with this co-velocity $\hat{v}$; we'll call it *paraframe* of S^*. One may notice that, if $\mathbf{u}'$ is the transformed velocity from $\mathbf{u}$ to S', then

$$\mathbf{u}^*_x = -\mathbf{u}'_x; \quad \mathbf{u}^*_y = -\mathbf{u}'_y; \quad \mathbf{u}^*_z = -\mathbf{u}'_z; \quad \Rightarrow \quad \mathbf{u}^* = -\mathbf{u}'.$$

In the case of the space-time 4-vector, we'll get

$$x^* = x', \quad y^* = y', \quad z^* = z', \quad t^* = -t'.$$

This outcome, along with length contraction and time dilation

$$\begin{cases} \Delta x = \frac{\alpha}{\beta}\Delta x^* \quad \Rightarrow \quad \Delta x = \sqrt{1-(1/\beta)^2}\Delta x^* \\ \Delta t = -\frac{\beta}{\alpha}\Delta t^* \quad \Rightarrow \quad \Delta t = -\frac{1}{\sqrt{1-(1/\beta)^2}}\Delta t^*, \end{cases} \tag{2.18}$$

this is $\Delta x^* = \Delta x'$ and $\Delta t^* = -\Delta t'$, clearly establish a *time reversion* between bradyonic and pseudotachyonic frames of coordinates. This, in turn, reveals the existence of *two opposite time flows in Nature*, an extraordinary – and full of consequences – feature of the Universe [2].

First of all, this implication of the transformation $S \leftrightarrow S^*$ allows us to finally understand why classic theories are essentially reversible in time. Further, it explains why there must be *conservation principles* such as, for instance, those applying to energy and linear impulse; in fact, these are not 'principles', they directly result from the possible inversion in the measure of time, concerning interactions, and the relativistic equivalence of all frames of coordinates.

2.2 Energy, Linear Momentum and Mass

Applying the transformation rule (2.6) and (2.7) to the transformation of the energy-momentum 4-vector

$$(iE/c,\, p_x,\, p_y,\, p_z)\,,$$

as well as the conditions (2.5), which in this case [for $\beta > 1$] correspond to

$$\begin{cases} i.c.\left(E^*/c^2\right) = -p''_x \\ p^*_x = -ic\left(E''/c^2\right), \end{cases} \tag{2.19}$$

we'll obtain the energy-momentum pseudotachyonic transformation rules:

$$\begin{cases} E^* = \frac{p_x c - E\beta}{\alpha} \\ p^*_x = \frac{p_x\beta - E/c}{\alpha} \\ p^*_y = p_y \\ p^*_z = p_z. \end{cases} \quad \text{and, identically,} \quad \begin{cases} E = \frac{p^*_x c - E^*\beta}{\alpha} \\ p_x = \frac{p^*_x\beta - E^*/c}{\alpha} \\ p_y = p^*_y \\ p_z = p^*_z. \end{cases} \tag{2.20}$$

If, in particular, the movement is parallel to the xx axis, this equation's system reduces to

$$\begin{cases} E^* = \frac{pc - E\beta}{\alpha} \\ p^* = \frac{p\beta - E/c}{\alpha} \end{cases} \quad \text{and} \quad \begin{cases} E = \frac{p^* c - E^*\beta}{\alpha} \\ p = \frac{p^*\beta - E^*/c}{\alpha}\,. \end{cases} \tag{2.21}$$

From (2.20) we see that, S' being the *paraframe* of S^*:

$$\begin{cases} E^* = -E' \\ \mathbf{p}^* = \mathbf{p}'\,. \end{cases} \tag{2.22}$$

This leads to the conclusion that pseudotachyonic transformations are fundamentally characterized by the conjugated phenomena of *time reversion* and *negative energies*. Besides, it implies that a *co-particle* is no more than the detection of an homologous tachyonic particle; in fact, the only way to detect it. And this means, as pointed out before, that there is no difference in nature between a particle and its co-particle: *the apparent difference is nothing but a relativistic effect due to its state of movement relatively to the reference frame that evaluates it* [2].

Following the previous reasoning, if a tachyonic particle $\mathbf{P}$ (immobile in its own frame, S'') moves with velocity v, the associated co-particle $\widehat{\mathbf{P}}$ – a sort of 'image' of the first (immobile in S^*) – must be detected moving with co-velocity

$$\hat{v} = \frac{c^2}{v}\,;$$

one can prove [see Appendix] that electric charge is anti-invariant in pseudotachyonic transformations and, so, that co-particles have opposite charges:

$$e = -e^*. \tag{2.23}$$

Incidentally, this equation explains the *universality of elementary charge*: electron's and proton's charges – that is, negative and positive elementary charges – have the same magnitude because the 'element of charge' is always the same. Besides, this charge conjugation, together with time and velocity reversion, brings forth and fully justifies the existence of CPT symmetry implicit in Lorentz transformations. Shouldn't it be so and all frames wouldn't be equivalent, violating the most fundamental of the principles of Relativity.

Remark that if we make $p^* = p_0^* = 0$ in (2.21), we obtain the energy E and the linear momentum p for the co-particle $\widehat{\mathbf{P}}$ in S as a function of its rest energy measured in S^*:

$$E = -\tfrac{\beta}{\alpha} E_0^* \quad \text{and} \quad p = -\tfrac{E_0^*}{c\alpha}, \tag{2.24}$$

where

$$\frac{\beta}{\alpha} = \frac{1}{\sqrt{1-\widehat{\beta}^2}} = \widehat{\gamma}, \tag{2.25}$$

Also, in order to respect the equivalence of all reference frames (bradyonic, tachyonic or pseudo-tachyonic), we'll assume in S^* the validity of Einstein's fundamental equation

$$E = mc^2 \quad \Leftrightarrow \quad E^* = m^* c^2.$$

One may wonder: why shouldn't it be $E^* = -m^* c^2$, as in Dirac theory? The answer is that this last equality conduces to a contradiction with $E' = m' c^2$ in the paraframe S'. As a matter of fact, from (2.22),

$$\begin{aligned} \mathbf{p}^* = \mathbf{p}' \quad \Rightarrow \quad & m^* \mathbf{u}^* = m' \mathbf{u}' \\ & m^* \left(-\mathbf{u}'\right) = m' \mathbf{u}' \\ & -m^* = m'; \end{aligned}$$

so, if $E^* = -m^* c^2$, it would be $E^* = E'$, which is false still according to (2.22). However, using the same reasoning, we conclude that, for Dirac's $E' = -m' c^2$, it must be $E^* = -m^* c^2$. We'll say that the *mass-energy compatibility* is invariant under whatever Lorentz transformation [see subsection 5.4].

As a result of (2.24) the mass of the co-particle results negative:

$$m = -\frac{\beta}{\alpha}\, m_0^* = -\frac{m_0^*}{\sqrt{1-\hat{\beta}^{\,2}}}. \tag{2.26}$$

Generally speaking, if m^* is the mass of a particle $\mathbf{P}$ moving in S^* with velocity u^* (parallel to the x^* axis), then the mass of its co-particle $\widehat{\mathbf{P}}$ in S is given by:

$$\begin{aligned} m = \tfrac{E}{c^2} = \tfrac{p^* c - E^* \beta}{\alpha c^2} = \tfrac{m^* u^* c - m^* c^2 \beta}{\alpha c^2} \\ \Rightarrow \quad m = -m^* \, \tfrac{\beta - u^*/c}{\alpha}. \end{aligned} \tag{2.27}$$

The kinetic energy of the co-particle also results negative:

$$\begin{aligned} E_k \quad &= \textstyle\int_0^{\hat{v}} F_T \, ds = \int_0^{\hat{v}} \frac{d}{dt} \left(m\hat{v}\right) \, ds \\ &= \left(1 - \tfrac{1}{\sqrt{1-(\hat{v}/c)^2}}\right) m_0^* c^2\,, \end{aligned} \tag{2.28}$$

E_0^* and m_0^* being the (supposed positive) rest energy and rest mass of the co-particle in its proper frame S^*. We may write (2.28) as $E_k = E_t + m_0^* c^2$, considering $E_t = mc^2$ the total energy of the co-particle in S and m its mass. If we now suppose $\hat{v} = 0$, that is to say $E_k = 0$, we obtain the equation for the rest energy, measured in S, of the co-particle $\widehat{\mathbf{P}}$:

$$E_0 = -m_0^* c^2 = -E_0^*, \tag{2.29}$$

which is compatible with (2.24). On the other hand, $E_0 = m_0 c^2$ and therefore

$$m_0 = -m_0^* \, ; \qquad (2.30)$$

thus, we preserve the usual equation, writing

$$E_k = (m - m_0)\, c^2 = E_t - E_0.$$

Intrinsically, this means that the proper energy and the proper mass of a particle $\mathbf{P}$ and of its homologous co-particle $\widehat{\mathbf{P}}$ have positive values for the first and negative ones for the second but exactly the same in modulus.

Furthermore, because of its negative mass/energy, co-particles have this remarkable dynamic characteristic:

> *The linear momentum vector* $\mathbf{p}$ *and the velocity vector* $\mathbf{u}$ *have opposite orientations.*

Related to this, it's not difficult to prove that if a massive co-particle, immobile at instant $t_0 = 0$, is submitted to a constant force $\mathbf{F}$ in the direction of the xx axis, its velocity at moment t is given by

$$u = c.\frac{(F/m_0 c)\, t}{\sqrt{1 + (F/m_0 c)^2\, t^2}}. \qquad (2.31)$$

One may see that this is the usual equation (for particles); however, in this case, owing to the *negative mass* of the co-particle, the velocity u will also be negative (opposite to the force) – but exactly symmetrical to the velocity of a particle under equal circumstances. Symmetrically, too, remark that

$$\lim_{t\to\infty} u = -c.$$

Hence, *if we push a co-particle forward, it will go backwards* – and this is a crucial feature regarding the behaviour of co-particles in interactions, including any kind of field, such as the gravitational field, as I pointed out in [3].

2.3 Waves, Photons and Co-photons

Once again, in order to respect the equivalence of all reference frames, PtR theory assumes the universal validity of Planck's fundamental equation:

$$E = h\nu \quad \text{and} \quad E^* = h\nu^*.$$

This equation and the preceding arguments imply that negative energies correspond to negative frequencies and vice-versa. We see then that a *co-photon* – the former *PtR antiphoton* – appears as a special case of co-particle; it corresponds to the detection in S of a particle detected as a photon in a pseudotachyonic frame S^*. It moves in our frame with co-velocity $\hat{c} = c$, in modulus, like a photon, but its energy (or, equivalently, its frequency ν) is negative, its linear momentum is opposed to the velocity vector and it must behave in an opposite way photons do in interactions with matter, co-matter or antimatter (for instance, regarding the Compton effect [2] or gravitational fields). So, contrary to what canonical theories assume for *antiphotons*, photons and co-photons are not equivalent.

From the quantum relations

$$E = h\nu = \hbar\omega \quad \text{and} \quad \mathbf{p} = \hbar\,\mathbf{k},$$

we obtain the phase 4-vector $(i\omega/c,\ k_x,\ k_y,\ k_z,)$ and, together with equations (2.20), the reverse transformation tables

$$\begin{cases} \omega^* = \frac{k_x c - \omega\beta}{\alpha} \\ k_x^* = \frac{k_x\beta - \omega/c}{\alpha} \\ k_y^* = k_y \\ k_z^* = k_z. \end{cases} \quad \text{and} \quad \begin{cases} \omega = \frac{k_x^* c - \omega^*\beta}{\alpha} \\ k_x = \frac{k_x^*\beta - \omega^*/c}{\alpha} \\ k_y = k_y^* \\ k_z = k_z^*. \end{cases} \qquad (2.32)$$

We may easily conclude that, if S' is the paraframe of S^*, then

$$\begin{cases} \omega^* = -\omega' \quad \text{or} \quad \nu^* = -\nu' \\ \mathbf{k}^* = \mathbf{k}'; \end{cases} \qquad (2.33)$$

this means, since $u_\varphi = \omega/k = \nu/\lambda$ is the phase velocity of the wave, that $u_\varphi^* = -u_\varphi'$ (which, as we have seen, is true for any velocity). The negative value for the transformed frequency ν^* (or for ν if we consider $\nu^* > 0$) relates to the pseudotachyonic inversion of time; it corresponds to a certain number of vibrations per -1 second.

I must correct here a silly mistake I made in [2] when I enounced that, in pseudotachyonic transformations, the phase $\varphi = kx - \omega t$ for a wave propagating along the xx axis is anti-invariant; as a matter of fact, *the phase is always invariant:*

$$\begin{aligned}\varphi^* &= k^*x^* - \omega^*t^* = \\ &\tfrac{k\beta-\omega/c}{\alpha}\cdot\tfrac{x\beta-ct}{\alpha} - \tfrac{kc-\omega\beta}{\alpha}\cdot\tfrac{x/c-\beta t}{\alpha} = \\ &\tfrac{1}{\alpha^2}\left[\left(kx\beta^2 - kx\beta t - x\beta\omega/c + \omega t\right) - \left(kx - kx\beta t - x\beta\omega/c + \omega\beta^2 t\right) = \right. \\ &\tfrac{1}{\alpha^2}\left[kx\left(\beta^2-1\right) + \omega t\left(1-\beta^2\right)\right] = \\ &kx - \omega t = \varphi.\end{aligned}$$

We should also conclude for this invariance by remarking that, if S' is the paraframe of S^*, then

$$\begin{aligned}\varphi^* &= k'x' - (-\omega')(-t') = \\ &k'x' - \omega't' = \varphi' = \varphi.\end{aligned} \tag{2.34}$$

Such an invariance is important in what concerns the subject of the next section.

We'll finish this paragraph with a note on Doppler effect. Consider a photon emitted by a tachyonic source (with velocity v). Following [8] but introducing the imaginary unity for the coordinate 0, *"a beam of coherent, monochromatic light can be characterized by the (null) wave 4-vector*

$$k^a = \left(i\tfrac{\omega}{c},\, \mathbf{k}\right), \quad \textit{in its contravariant form,} \quad k^a = \left(i\tfrac{\omega}{c},\, k_x,\, k_y,\, k_z\right),$$

(...) and *related to the four-momentum as follows:*

$$p^a = \hbar k^a = \left(i\frac{E}{c},\, p_x,\, p_y,\, p_z\right)''.$$

Applying the pseudotachyonic Lorentz matrix, (2.11), to the transformation of the wave vector $[\mathbf{k}^* = \mathbf{\Lambda}^*\mathbf{k}]$, we'll get for the $a = 0$ component (θ being the direction cosine, $k^1 = k^0\cos\theta$):

$$\begin{aligned}i\tfrac{\omega^*}{c} &= -\tfrac{\beta}{\alpha}\left(i\tfrac{\omega}{c}\right) - \tfrac{i}{\alpha}k^1 \\ &= -\tfrac{i}{c}\tfrac{\beta}{\alpha}\,\omega\left(1 - \tfrac{\cos\theta}{\beta}\right);\end{aligned}$$

so, making $\widehat{\beta} = 1/\beta$ for the bradyonic detection velocity of the source, this is, $\frac{\beta}{\alpha} = \widehat{\gamma}$, we finally obtain

$$\frac{\omega}{\omega^*} = \frac{\nu}{\nu^*} = -\frac{\sqrt{1-\widehat{\beta}^2}}{1-\widehat{\beta}\cos\theta}. \tag{2.35}$$

Making now in this equation $\theta = \pi$ (source moving directly away from the observer) or $\theta = 0$ (source moving straight towards the observer) and, on the other side, $\theta = n/2\pi$ (source moving in a right angle towards the observer), we'll get the expressions for respectively the *longitudinal* and the *transversal pseudotachyonic Doppler effects*:

$$\nu = -\nu^*\sqrt{\tfrac{c\mp\hat{v}}{c\pm\hat{v}}}\,; \quad \text{and} \quad \nu = -\nu^*\sqrt{1-(\hat{v}/c)^2}, \tag{2.36}$$

using the upper signs if the source is moving away from the observer, with a detection velocity $\hat{v} = c^2/v$; the lower ones if the source is coming closer. Both the results are symmetrical to those obtained for the bradyonic Doppler effect. They both mean that the photon is detected as a co-photon, with negative energy.

Remark that one cannot really distinguish radiation from co-radiation as far as wavelength is concerned because wavelength, supposed positive in S^*, will also be positive in our frame S. In one case or the other, the longitudinal and transversal Doppler effects are expressed by

$$\lambda = \lambda^*\sqrt{\tfrac{c\pm\hat{v}}{c\mp\hat{v}}} \quad \text{and} \quad \lambda = \tfrac{\lambda^*}{\sqrt{1-(\hat{v}/c)^2}}\,. \tag{2.37}$$

However a beam of co-radiation, because of its negative energy, should be deviated by a material gravitational field inversely to a beam of radiation [2].

2.4 De Broglie Waves

This is a classic: in his 1924 PhD thesis, Louis de Broglie applied to massive particles the same Plank's equation $E = h\nu$ established for the light and therefore the same dualistic wave-particle nature and behaviour: to a free particle moving with constant velocity v (supposing $|v| < c$) must correspond a monochromatic plane wave propagating in space, in the same direction of the particle's motion, which phase is a function of the position vector $\mathbf{r}$ and time t:

$$\psi(x,t) = Ae^{i(\mathbf{kr}-\omega t)} = Ae^{i/\hbar(\mathbf{pr}-Et)}. \tag{2.38}$$

If the particle moves along the xx axis or parallel to it, the phase is $\varphi = kx - \omega t$ and the wave function becomes

$$\psi(x,t) = Ae^{i(kx-\omega t)} = Ae^{i/\hbar(px-Et)}. \tag{2.39}$$

Remark that, in this case and as a result of the phase and amplitude invariance, the wave equation for this free particle remains the same in any other reference frame, for instance a pseudotachyonic one and its paraframe:

$$\psi(x,t) = \psi'(x,t) = \psi^*(x^*,t^*); \qquad (2.40)$$

however, energy and time are reversed from S' to S^*, in which the particle appears as an homologous co-particle. On the other hand, if in S a particle $\mathbf{P}$ and an homologous co-particle $\widehat{\mathbf{P}}$ move with the same velocity (forward in time), since $\widehat{m} = -m$, their wave functions are, concerning the exponential part, the inverse one from the other:

$$\begin{aligned}\widehat{\psi}(x,t) &= Ae^{i/\hbar(\hat{p}x-\hat{E}t)} = Ae^{-i/\hbar(px-Et)} \\ &\Rightarrow \psi\widehat{\psi} = A^2 \\ &\Rightarrow \text{if } A=1, \quad \widehat{\psi}(x,t) = [\psi(x,t)]^{-1}.\end{aligned} \qquad (2.41)$$

This obviously corresponds to the interchange of space-time axis: $px \leftrightarrow Et$.

The crucial point now is that this de Broglie wave propagates with a phase velocity u_φ which turns out to be tachyonic:

$$u_\varphi = \frac{\partial\varphi}{\partial t} = \frac{\omega}{k} = \frac{mc^2}{mv} = \hat{v}. \qquad (2.42)$$

This was the major problem encountered by de Broglie himself and by subsequent authors, who saw in this extravagant phase velocity *"an artificial conception without physical signification"* [9]. I disagree and I analysed this issue in [3]; here, it's quite relevant to remark that, if the wave is tachyonic, it must be detected with its co-velocity

$$\hat{u}_\varphi = c^2/\hat{v} = v, \qquad (2.43)$$

which is the velocity of the particle itself. As I stated in [3], this is, I believe, too remarkable to be just a coincidence!

De Broglie tried to surround this *tachyonic problem* by admitting that, instead of associating a moving particle to a single 'pilot-wave', as his original idea suggested, we should associate it to a 'wave packet' in certain conditions [10]: a group of monochromatic waves, which frequency slightly differ from the frequency of the former wave and which, in fact, progresses with a group velocity equal to the velocity of the particle:

$$\begin{aligned}u_g &= \tfrac{\partial\omega}{\partial k} = \tfrac{\partial\omega}{\partial\beta} \big/ \tfrac{\partial k}{\partial\beta} = \\ &\tfrac{m_0c^2}{\hbar}\tfrac{\beta}{(1-\beta^2)^{3/2}}\left[\tfrac{m_0c}{\hbar}\tfrac{1}{(1-\beta^2)^{3/2}}\right]^{-1} = \\ &c\beta = v,\end{aligned}$$

this result being compatible with the transport of energy associated to material particles. But the attempts to interpret elementary particles, such as electrons, as wave packets brought serious difficulties, starting with the dispersive nature of these packets [9].

From my point of view, the tachyonic wave has a real physical sense (and existence) and, according to (2.43), the wave packet is just a way of detecting it and so the correspondent transport of energy. Removing the classical 'tachyonic objection', it's quite evident that, in fact, the so-called *duality wave-particle* must not mean that wave and particle are the *same thing* (or even two *aspects* of the same thing). On the contrary, their velocities being different, we are dealing with *different entities* although intimately associated – and, for the sake of consistency, we may probably generalize this statement to photons.

Finally, if we apply the equation (2.39) to a tachyonic particle moving with velocity v (using the standard Lorentz transformations and keeping in mind that the imaginary value for m cancels in the fraction ω/k), we'll obtain for its de Broglie wave a phase velocity $\hat{v}$, which is precisely the subluminal velocity of the correspondent detected *co-particle*. That's why I suggested in [3] that the link between a tachyonic particle and its associated co-particle might be precisely its de Broglie wave.

2.5 De Broglie "Periodic Phenomenon" and Force Fields

In its proper frame, a particle has no wave associated to it but only what De Broglie himself called a "periodic phenomenon". But what may be the nature of this "internal vibration", which is not a mechanical process, for a particle at rest?

I will not develop this issue here, just point out some notes, taken back from [3] with some updates and corrections. It is appropriate to do it because of the controversial aspects of negative mass and energy. In [11], Louis De Broglie introduced *"the idea according to which the particle can be likened to a small clock of frequency* $\nu_0 = E_0/h$*"*. I presented myself the idea that this De Broglie's "periodic process" for a particle – its 'clocklike' behaviour – deeply relates to all the force fields the particle is the source of. More precisely, we'll consider – as in Quantum Mechanics – that every interaction between particles is mediated by certain *mediator particles*. I will not discuss here the concept and theoretical use of 'virtual particles' called into service as mediators, which may seem absurd since physicists conveniently allow to these extraordinary particles characteristics and behaviours they do not – under no circumstances! – admit to 'actual particles', and yet not violating Physics profound beliefs: *"for example, they progress backwards in time, do not conserve energy, and travel faster than light. That is to say, looked at one by one, they appear to virtually violate basic laws of physics. Actual particles of course never do so. (...) Many physicists believe that, because of its intrinsically perturbative character, the concept of virtual particles is often confusing and misleading, and is thus best avoided"* [12]. But I'll keep the unifying idea of mediator particles simply because it is deeply logical and reasonable; however, according to PtR, these mediators may have positive or negative energies. Under this point of view, I propose that it's also reasonable to admit the following hypothesis:

1. the spontaneous emission of all the mediators by a source particle obeys to its *internal vibration*, with *inner frequency* $\nu_\odot = \nu_0 = m_0c^2/h$.
2. the whole of mediators emitted in a *inner time-lapse* $\tau_\odot = 1/\nu_\odot = h/E_0$ by a massive particle – that is to say, the whole of all interactions the particle is the source of – must obey to an *energetic equilibrium globally null*. This is due to the law of conservation of energy applied to the particle itself and has this profound consequence that both *positive* and *negative-energy* mediator particles must be emitted, thus creating positive and negative-energy distinct fields. It seems also to imply the conclusion that a single universal force is impossible. Moreover, because of the necessary conservation of the linear momentum for the particle itself, *each mediator is probably emitted in pairs and in opposite directions*.
3. in the proper frame of the particle, the mediators, for each field, have always the same energy.

Further on, we'll make some reflections about the notions of "emission" (right ahead) and "proper frame" of a particle (subsection 5.4). We'll assume then that any force field – operating through the transmission of energy and linear momentum carried by its mediator particles – is characterized either by positive either by negative energy; therefore, for now, we'll classify them as *positive* (i.e., repulsive) or *negative* (i.e., attractive) fields. Moreover, we'll classify all kinds of particles as follows:

- as the source of a certain field, as *neutral*, *positive* or *negative* ones;
- regarding their reaction to a certain field, as *neutral*, *pro-reactive* or *anti-reactive* ones.

Remark that, in this conception, the attractive or repulsive nature of a field doesn't exactly lie in the direction of the resulting movement of a particle submitted to it (the phenomenological approach to the problem expressed by Newton's and Coulomb's laws) but in the linear momentum the field induces. In case of *positive mediator particles* (ϵ^+) this linear momentum has the same direction of the mediator movement; in case of *negative mediator particles* (ϵ^-), it is contrary to its movement. This is why the correspondent fields result respectively repulsive or attractive. Furthermore, something in the nature of *anti-reactive particles* to a certain field respond to this field like co-matter ordinarily does: for them velocity (or force) and linear momentum have opposite senses and so they react negatively. However, we must not conceive an anti-reactive particle as necessarily a co-particle; either a particle or a co-particle can be in one of the categories listed above. We must also be aware that a positive (or a negative) field doesn't necessarily have a pro-reactive (or an anti-reactive) particle as its source.

For instance, the gravitational field generated by a material *particle* ($m > 0$) is *attractive*, a negative field, this meaning that the *graviton*, its hypothetical mediator, should be a *co-particle*. It carries a linear momentum opposed to its movement; when reaching a particle, the graviton transfers to it (to make it simple) a part of that linear momentum. If the hit particle has positive mass/energy (which is the case of ordinary *pro-reactive* matter), the transferred momentum will roughly – statistically – make it move in the direction of the source of the field. The result is the opposite in the case of a co-particle target, because its negative mass/energy makes it an *anti-reactive* particle: it will move away from the source.

The situation concerning the gravitational field created by *co-matter* ($m < 0$) is precisely the inverse. In a first approach, the pseudotachyonic transformation of the previous scenario seems to make it clear that this field is due to the emission of *antigravitons* – which are positive particles, with positive energy. In this case, the linear momentum has the same direction of the mediator particle movement, causing the field to be *repulsive*. A particle (pro-reactive) will respond to it driving away from the source; a co-particle (anti-reactive) will do it negatively, approaching it.

Notice that the situation is actually a little more complicated than this because of the reversion of time. We'll see no co-particle emitting mediator particles (in the case, antigravitons) but, on the contrary, absorbing them in an incoming flux. On the one hand, this corresponds to a negative "emission", due to a negative inner time-laps $\tau_{\odot}$; on the other hand, this epistemological point of view is perfectly compatible with the concept of Faraday's line of force, except that these should be inverted with the sign of the electric charge of the source because of Franklin's 'mistake' in attributing those signs (as referred ahead). Amazingly, because of the conservation of momentum and energy, the result of this "negative emission" is exactly the same!

To make a summary, we'll say that:

1. a massive particle creates an attractive gravity field; a massive co-particle creates a repulsive one;
2. a particle reacts positively to a field; a co-particle reacts negatively to it.

From these assertions we can conclude for the following generic *observable effects* concerning gravitational interaction:

- two particles attract each other;
- two co-particles also attract each other;
- a particle and a co-particle repel each other.

Generally talking, the creative and reactive status of a particle relatively to a field depends of course on itself and on the concerned field. That's why we'll assign to a material generalized particle two *charge factors* equal to ± 1: a creative factor ϕ^C_{field} and a reactive factor ϕ^R_{field}. These charge factors define a *creative energy* E^C_{field} and a *reactive energy* E^R_{field} given by

$$E^C_{field} = \phi^C_{field} \left| E^C \right| \quad \text{and} \quad E^R_{field} = \phi^R_{field} \left| E^R \right|, \tag{2.44}$$

which are the relevant 'energies' concerning the particle's interactions under the field. Note that we must also draw the concept of a particle's *element of charge* as the element *within* the particle responsible for the creation of the field, which is not necessarily the whole particle; E^C refers to this element of charge and E^R to the particle itself because it always reacts as a whole.

We may write the above equation in terms of masses, in the condition that we put

$$E = \diamond mc^2,$$

where $\diamond$ is the *alignment factor* [see subsection 5.3]; $\diamond = 1$ for particles and co-particles; $\diamond = -1$ for Dirac's prime-antiparticles (generalizing his negative-energy electrons) and co-prime-antiparticles (their pseudotachyonic transformation):

$$m^C_{field} = \diamond\, \phi^C_{field} \left|m^C\right| \quad \text{and} \quad m^R_{field} = \diamond\, \phi^R_{field} \left|m^R\right|. \tag{2.45}$$

It's easily understandable that both ϕ^C_{field} as ϕ^R_{field} change sign under pseudotachyonic transformations.

Concerning gravity, as we have seen, for particles and co-particles ($\diamond = 1$) we'll have $\left|E^C\right| = \left|E^R\right| = |E|$ as well as respectively $\phi^C_{field} = \mp 1$ and $\phi^R_{field} = \pm 1$; therefore,

$$\begin{cases} m^C_g = -\left|m\right| \\ m^R_g = \left|m\right| \end{cases} \text{for particles; and} \quad \begin{cases} m^C_g = \left|m\right| \\ m^R_g = -\left|m\right| \end{cases} \text{for co-particles.}$$

But this means that we may simply write (remembering that co-particles have negative mass):

$$\begin{cases} m^C_g = -m \\ m^R_g = m. \end{cases} \tag{2.46}$$

The situation described above is more or less the same concerning electrostatic fields. But in this case, the field created by an electron is mediated by *photons*, which have positive energy; it is a positive, *repulsive* field. This hypothesis comes from a simple reasoning based on Compton effect. But it means that, unfortunately, Benjamin Franklin made a mistake in identifying the term “positive” with vitreous electricity and ”negative” with resinous electricity. He should have done the inverse!

We'll assign now to a material particle, in equations (2.44) and (2.45) a single electrostatic charge factor $\phi_{el} = \phi^C_{el} = \phi^R_{el}$ given by

$$\phi_{el} = 0 \quad \text{for } q = 0; \quad \phi_{el} = -\tfrac{q}{|q|} \quad \text{for } q \neq 0, \tag{2.47}$$

where q is the electric charge of the particle (the $-$ sign resulting from Franklin's ‘mistake'). As a consequence, we'll have

$$\begin{cases} E^C_{el} = \phi_{el} \left|E^C\right| \quad \text{and} \quad E^R_{el} = \phi_{el} \left|E^R\right|; \text{ or} \\ m^C_{el} = \diamond\phi_{el} \left|m^C\right| \quad \text{and} \quad m^R_{el} = \diamond\phi_{el} \left|m^R\right|. \end{cases} \tag{2.48}$$

In short, apart from neutral particles $\left(\phi_{el} = m^C_{el} = m^R_{el} = 0\right)$ and restricting for now the problem to positive alignment factor (particles and co-particles), this means that:

1. particles with negative charge create positive fields and are pro-reactive;
2. particles with positive charge create negative fields and are anti-reactive,

this hypothesis conducing to the *observable effects* expressed by Coulomb's Law:

- two particles with equal charges repel each other;
- two particles with opposite charges attract each other.

In the case of *electrons* and *co-electrons*, which are elementary particles, we'll have $m^C = m^R = m$ in both cases and respectively $\phi_{el} = \pm 1$; that is to say, $m^C_{el} = m^R_{el} = m$. This means that another electron reacts positively to the field of an electron, moving away from it, whilst instead a co-electron, an anti-reactive particle, reacts negatively approaching the source. Inversely, the field created by a co-electron is attractive (mediated by co-photons). An electron reacts positively, approaching the source, whilst a co-electron reacts negatively, moving away from it.

In what comes to the electrostatic field produced by a *proton*, only its element of charge (should we say, the *‘positron within it'*) is relevant, not the total mass of the particle; but, of course, in its reaction to another electrostatic field the total mass is relevant. So, we'll consider $\phi_{el} = -1$

and $m^C_{el} = \hat{m}$, the mass of the co-electron; in the reaction to another electrostatic field, we must consider as relevant $m^R_{el} = -M$, the symmetrical of the mass of the proton, because 'all' the particle reacts to the induced momentum.

The reasoning for a *co-proton* [$\phi_{el} = 1$; $m^C_{el} = m$; $m^R_{el} = M$] is self-evident: it creates a repulsive field and reacts positively to other electrostatic fields.

Finally, in terms of numbers: for instance, $\tau_\odot = 1/\nu_0 = 8.09 \times 10^{-21} s$ for the electron and $\tau_\odot = 4.41 \times 10^{-24} s$ for the proton. Following the idea that the spontaneous emission of all the mediators by a source particle obeys to its *element of charge internal vibration*, the number N of mediator particles of a certain field emitted in a time-laps t, in the particle's proper frame, is given by

$$N = 2\nu_0 t, \quad \text{which means that} \quad \tfrac{dN}{dt} = 2\nu_0. \tag{2.49}$$

During this time-laps (negative for $E_0 < 0$), each mediator ϵ covers a distance $r = v_\epsilon\, t$ (so, $dr = v_\epsilon dt$). In the case of gravitational and electrostatic interactions, the mediator mass is null and its velocity is $v_\epsilon = c$; then $r = ct$ or $dr = c\, dt$. Since the volume of the spherical sector between radius r and $r + dr$ is given by

$$\begin{aligned} dV &= \tfrac{4}{3}\pi \left[(r + dr)^3 - r^3\right] \\ &= \tfrac{4}{3}\pi \left[3dr \cdot r^2 + 3\,(dr)^2 \cdot r + (dr)^3\right], \end{aligned}$$

considering $r >> dr$, we get

$$dV \approx 4\pi r^2\, dr = 4\pi c r^2\, dt \quad \Rightarrow \quad \tfrac{dV}{dt} \approx 4\pi c r^2.$$

Therefore, the mediators density in this sector is:

$$\frac{dN}{dV} = \frac{dN}{dt} \Big/ \frac{dV}{dt} = \frac{\nu_0}{2\pi c r^2}$$

or

$$\frac{dN}{dV} = \frac{E^C_0}{2\pi h c r^2} = \frac{\diamond m^C_0\, c}{2\pi h r^2}, \tag{2.50}$$

where $\diamond$ is again the *alignment factor* and m^C_0 the creative inertial mass of: 1) the source particle in the case of a gravitational field; 2) the element of charge in the case of an electrostatic field.

This mediators density (where we find the relativistic and quantum constants, c and h respectively) closely relates to the Gauss flux theorem and leads us – as long as Euclidean geometry remains valid – to a reformulation of Newton and Coulomb laws. Besides, the equation above is the reason why both these laws are formally identical, the phenomenological result of similar discrete interactions.

As a collateral effect, all this reasoning justifies Einstein's principle of equivalence between *gravitational and inertial masses*, in the basis of General Relativity, which isn't really a 'principle' but a consequence; in fact, both masses are the same, the one and only inertial mass. But it also implicates that the distortion of space-time by a gravitational field isn't really 'objective': this is, for a particle submitted to it, it depends on the particle be pro-reactive or anti-reactive; in one case or the other, the 'subjective' distortion appears reversed. It seems that Einstein took here the effect for the cause; this is what I call "Einstein's mistake", which has profound consequences, far beyond the scope of this article.

3 GENERALIZED DIRAC THEORY OF THE ELECTRON

3.1 Negative-energy Solutions

I intend to re-examine and discuss now the ingenious Dirac equation for the electron, the basis for the subsequent standard approach to antimatter. It is well known that in 1929, in his *"Theory of Electrons and Protons"*, Dirac announced a difficulty in his audacious theory: for the first time in Physics, strange negative energies appeared as solutions for an equation, solutions he could not discard. It's important to fully understand how negative energy arises in Dirac theory of the electron... and how it has been eluded since then. In §67 of [13], his book on Quantum Mechanics, he remarks that the relativistic Hamiltonian in classical mechanics is given by

$$H = c \left(m^2 c^2 + \sum_{j=1}^{3} p_j^2 \right)^{1/2}, \tag{3.1}$$

this leading to the wave equation

$$\left[p_0 - \left(m^2 c^2 + p_1^2 + p_2^2 + p_3^2 \right)^{1/2} \right] \psi = 0, \tag{3.2}$$

where the $p's$ are interpreted as operators, as in the Schrödinger representation (making $p_0 = H/c$). He explains then that this equation is unsatisfactory *"because it is very unsymmetrical between p_0 and the other $p's$"*. The 'obvious' solution is to multiply the equation (3.2) by the operator

$$\left[p_0 + \left(m^2c^2 + p_1^2 + p_2^2 + p_3^2\right)^{1/2}\right]\psi = 0, \quad (3.3)$$

in order to finally obtain a relativistic invariant equation:

$$\left[p_0^2 - m^2c^2 - p_1^2 - p_2^2 - p_3^2\right]\psi = 0. \quad (3.4)$$

But Dirac immediately alerts to the fact that *"equation (3.4) is not completely equivalent to equation (3.2) since, although every solution of (3.2) is also a solution of (3.4), the converse is not true. Only those solutions of (3.4) belonging to positive values for p_0 are also solutions of (3.2)"*. As a matter of fact, the multiplying operator (3.3) corresponds to the negative Hamiltonian

$$H = -c\left(m^2c^2 + \sum_{j=n}^{3} p_j^2\right)^{1/2}.$$

In search of a rational and linear equation in all p_j, Dirac arrives to his famous equation, in the form

$$\left[p_0 - \alpha_1 p_1 - \alpha_2 p_2 - \alpha_3 p_3 - \alpha_m mc\right]\psi = 0, \quad (3.5)$$

where the function wave is no longer a scalar but a vector ψ with four components, known as *Dirac spinor*, and the four coefficients α_i and α_m are 4×4 matrices.

Remark that Dirac implicitly assumes *positive values* for the rest mass m in (3.5), an equivalent equation to (3.4) which he claims to be the *"correct relativistic wave equation for the motion of an electron in the absence of a field. This gives rise to one difficulty, however, owing to the fact that (3.5), like (3.4), is not exactly equivalent to (3.2), but allows solutions corresponding to negative as well as positive values of p_0"*.

This means that Relativity itself obliges to consider negative-energy solutions for the electron. Following the classic rule, Dirac then generalizes the equation (3.5) to the case when there is an electromagnetic field present:

$$\left[\left(p_0 + \frac{e}{c}A_0\right) - \sum_{j=1}^{3}\alpha_j\left(p_j + \frac{e}{c}A_j\right) - \alpha_m mc\right]\psi = 0, \quad (3.6)$$

where A_0 and $\mathbf{A}$ are the scalar and vector potentials of the field at the point where the electron is. This, he says, *"is the fundamental wave equation of the relativistic theory of the electron"*. Further on, in §73, he states that *"the wave equation for the electron admits of twice as many solutions as it ought to* [the double solution of Pauli's spin theory], *half of them referring to states with negative values for the kinetic energy $cp_0 + eA_0$. This difficulty was introduced as soon as we passed from equation (3.2) to (3.4) and **is inherent in any relativistic theory*** [The bold is mine]. *It occurs also in classical relativistic theory, but is not then serious since, owing to the continuity in the variation of all classical variables",* a kinetic energy cannot subsequently be negative.

The inherent difficulty Dirac talks about is the subject of PtR, where the discontinuity between positive and negative energies clearly appears in correlation to mutually pseudotachyonic frames of coordinates, that is, to a discontinuity/opposition of two time flows. But, of course, ignoring this hypothesis, Dirac continues in his own field: *"In the quantum theory, however, discontinuous transitions may take place, so that if the electron is initially in a state of positive kinetic energy it may make a transition to a state of negative kinetic energy. It is therefore no longer permissible simply to ignore the negative-energy states, as one can do in classic theory"*.

He then examines the negative solutions more closely. He judiciously do this (and he can do it because the four $\alpha's$ all mutually anticommute and the square of each is the unity) by interchanging the expressions for α_2 and α_m in such a way that all the elements of the matrices representing α_1, α_2 and α_3 are real and all those of the matrices representing α_m are pure imaginary or zero. Then, putting $-i$ for i all trough the new matrix equation, and remembering that $p_j = i\hbar\frac{\partial}{\partial x^j}$, he gets

$$\left[\left(p_0 - \frac{e}{c}A_0\right) - \sum_{j=1}^{3}\alpha_j\left(p_j - \frac{e}{c}A_j\right) - \alpha_m mc\right]\bar{\psi} = 0. \tag{3.7}$$

"Thus each solution ψ of the wave equation (3.6) has for its conjugate complex $\bar{\psi}$ a solution of the wave equation (3.7). Further, if the solution ψ of (3.6) belongs to a negative value for $cp_0 + eA_0$, the correspondent solution $\bar{\psi}$ of (3.7) will belong to a positive value for $cp_0 - eA_0$. But the operator in (3.7) is just what one would get if one substituted $-e$ for e in the operator in (3.6). It follows that each negative-energy solution of (3.6) is the conjugate complex of a positive-energy solution of the wave equation obtained from (3.7) by substitution of $-e$ for e, which solution represents an electron of charge $+e$ (...) moving through the given electromagnetic field. (...)

In this way we are led to infer that the negative-energy solutions of (3.6) refer to the motion of a new kind of particle having the mass of an electron and the opposite charge. Such particles have been observed experimentally and are called positrons*"* [13] §73.

This is of pure genius! But the point now is: **negative energies** are in the very basis of the concept of standard antiparticles – and this is so even today – and co-particles as well. Therefore, negative energies shouldn't be a bone of contention between PtR and Quantum mechanics. But it is. Why? First of all, because, somehow or other, Quantum theory surrounds the problem: energy, after all, turns from negative to positive.

This began with Dirac correspondence above, $\psi \leftrightarrow \bar{\psi}$, and continued in his strange "hole" conjecture, designed to extricate himself from the embarrassment. In fact, he adds: *"We cannot, however, simply assert that the negative-energy solutions represent positrons, as **this would make the dynamical relations all wrong**. For instance, **it is certainly not true that a positron has a negative kinetic energy*** [Once again, the bold is mine]. *We must therefore establish the theory of the positrons on a somewhat different footing. We assume that nearly all the negative-energy states are occupied, with one electron in each state in accordance with the exclusion principle of Pauli. An unoccupied negative-energy state will now appear as something with a positive energy, since to make it disappear, i.e., to fill it up, we should add to it an electron with negative energy. We assume that* these unoccupied negative-states are the positrons.*"*

Despite his audacity, Dirac reasons in classical terms. PtR – with its negative masses and negative (kinetic) energies – implies the need to re-examine these issues, reviewing some concepts and dynamic relations and rewriting equations, for example on gravitational and electrostatic fields. Meanwhile and quite comprehensibly, Dirac himself thought about an *electron in a negative-energy state* as something totally strange to our experience. So, in his conception, the *positron* isn't this uncommon particle but its *absence*.

Though the concept of an endless sea of negative-energy electrons appears as a quite odd one, Dirac extracted from it the conclusion that an electron and an anti-electron should annihilate each other producing pure energy and, conversely, that energy could produce a pair of homologous particles. Remark that the equivalence of both processes – pair annihilation and its reverse, pair creation – is fully justified by PtR theory since they correspond to time reversion: the pair annihilation in a pseudotachyonic frame S^* appears as a pair creation in the bradyonic paraframe S' and vice-versa [2]. For that very reason (in fact the equivalence between S' and S^*) PtR also justifies Dirac statements on the *"symmetry between occupied and unoccupied fermion states"*.

However, it's hard to agree with several consequences of an endless sea of particles with negative energy – Dirac himself was well aware of them –, mainly the implication of *"a distribution of negative-energy electrons of infinite density everywhere in the world* [which] *does not contribute to the electric field"* [13] §73. This seems to be extraordinary and, anyway, doesn't make sense from the point of view of PtR: any vaccum region is the same for S' and S^*; so, the infinite distribution of negative-energy electrons in S' corresponds to an infinite distribution of positive-energy electrons in S^*, which is hard to sustain as not contributing to the electric field. This simply violates the principle of equivalence.

Let us focus now on the *conjugation method* followed by Dirac to link the equations (D7) and (D8). Applied to the 'prime-antiparticle' $\mathbf{e}^\bullet$ (this is, the negative-energy electron), the conjugate $\overline{\psi}$ represents an inversion of energy, but apparently not together with an inversion of time as in pseudotachyonic transformations. Now, there is no usual Lorentz transformation capable of relating negative-energy electrons to positive-energy anti-electrons, both with positive mass and in the same temporal direction. So, unlike pseudotachyonic transformations, which allow us to understand that a 'straight' particle and its homologous co-particle aren't but aspects of one single *'archeparticle'* [from the Greek word *arché*, meaning *beginning* or *origin*], it's hard to achieve the same goal regarding negative-energy electrons and positrons – or, worst, simple electrons and positrons; do they also correspond to an unique archeparticle?

A decisive step towards this last identification was taken by Richard Feynman, who, in a way, rid himself from negative-energy particles. In 1949 he discovered that the space-time description of a positron moving forward in time is exactly equivalent to the description of an electron moving backward in time [14]; this is valid for every pair of homologous particles. He copiously illustrated this conception in his famous diagrams, noting that photons are their own antiparticles.

Other authors understood that reversing both time and energy in equations simply allows to directly 'identifying' negative-energy electrons with positrons; and, by this procedure, once again *negative energy turns into positive*. It comes to replace the equation (3.7) by an equivalent one, relating the antiparticle $\overline{\mathbf{e}}$, not to the 'prime-antiparticle' $\mathbf{e}^\bullet$ but to the 'straight' particle $\mathbf{e}$. Though *"the original problem of a negative energy solution still remains"* [15], this conversion is, until today, the usual way to deal with this issue. See, for instance, the following explanation given for a solution of Dirac equation in [16]: *"By the same reasoning, the solution for u_p is* (...) *so that, in the limit $\mathbf{p} = 0$ and $E_0 = -mc^2$,*

$$\Psi(t) \rightarrow \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix} e^{imc^2 t/\hbar} \quad \text{or} \quad \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix} e^{imc^2 t/\hbar}$$

which describes particles moving backward in time. Thus, the interpretation is that the negative energy solutions correspond to anti-particles." Remark, however, that another valid interpretation is to consider, for negative-energy solutions in the above equations, particles moving forward in time but with negative mass.

The conversion time/energy works, so most people don't seem to worry about the subject; but it's quite curious that it seems easier to conceive a reversion of time – though this is also "totally strange to our experience" – than negative energies. This is because of Relativity, of course, and maybe because almost all equations in Physics are reversible in time, apparently in contradiction with a well-defined time-arrow. Nevertheless, the solution of Feynman and others mainly consists on a mathematical procedure, coherent with the CPT theorem, but it seems to be no basis in Quantum theory to justify time reversion. In a way, this lack of bedrock still keeps the debate alive on whether or not there is actually temporal reversion. For instance, in [17], the author declares: *"What I am saying is: the statement "positrons are backward going electrons" is a convenient and accurate mathematical representation for calculation purposes. "As if". There has not been an indication, not even a tiny one, that in nature as we study it experimentally anything goes backwards in time, as we define time in the laboratory."*; and, in [18]: *"To the best of my knowledge, most physicists don't believe that antimatter is actually matter moving backwards in time. It's not even entirely clear what would it really mean to move backwards in time, from the* *popular viewpoint.* (...) *Of course, since we can't actually reverse time, we can't test in exactly what manner this is true."*

I just want to make a brief comment on this complex issue. We don't 'see' time going backwards, in the same way we don't actually 'see' it going forward. Even if the human perception of time flowing is somehow mysterious but surely linked to memory, for an elementary particle there is no perception involved, there isn't even time. However, from the outside, there is a sequence of events, occurring in a time flux, associated with a time-arrow in whatever reference frame. In this World, as philosophers say, the past is already gone, the future isn't here yet; apparently, there isn't but the present, there isn't but the *instant*. This means that the coexistence of two temporal flows happens moment to moment; in this sense, it is not continuous. But, in another sense, there is a continuous superposition of instants – from two time-streams – 'going' in opposite directions; and it's just in each of these instants that the two streams meet in interactions. In this regard, I analysed in [1] the ideal experiment of a light beam sent from one extremity of a bar to the other and then reflected to the origin; in mutually pseudotachyonic reference frames, the experiment appears in a reversed order: the emission of the beam in one frame appears as the reception in the other, the reflex being common to both. The analysis can be extended to irreversible phenomena, such as those studied in Thermodynamics. In [2] I show that co-matter *"must have a distinctive characteristic of negative absolute temperatures"*. The second law of Thermodynamics is quite subtle here. Take the irreversibility in the transfer of heat by conduction or radiation; when two bodies initially of different temperatures come into thermal connection, then heat always flows from the hotter body ($\mathbf{B_1}$) to the colder one ($\mathbf{B_2}$). Let T_1 and $T_2 < T_1$ be the respective temperatures of the two bodies. In a PtR transformation, because of time reversion, we'll verify that heat flows from $\mathbf{B_2}$ to $\mathbf{B_1}$; the contradiction isn't but apparent because, in fact, both temperatures result negative in the paraframe S^* ($T_1^* = -T_1$ and $T_2^* = -T_2$) and so $T_2^* > T_1^*$. Therefore, heat keeps flowing from the hotter body to the colder one. However, still due to the reversal of time, this no longer represents a spontaneous progressive passage of a system from a more organized state to a more disorganized (i.e., homogeneous) one. In general, from a macroscopic point of view, we'll perhaps witness amazing events testifying time reversion, like an apple, instead of falling from a tree, flying back to the branch!... It is not guaranteed that we won't see it one day in this vast Universe... looking carefully!

Coming back, the reluctance to admit negative states for energy and, above all, for mass largely persist in the scientific community [and it's quite amusing to remember that, after Dirac published his seminal paper, there was quite an *"hostility towards these negative energy solutions."* [15]].

Now, in contrast with the referred *ad hoc* procedures, PtR theory *predicts* negative energy and time reversion, bringing forward the fundamental existence of two time flows; so, in a way, it legitimates those procedures. But there's a problem: the negative-energy electron in PtR – this is, the co-electron – isn't the same as the negative-energy electron in Dirac theory. For instance, a serious divergence point here between PtR and Quantum theory lies in the **mass**, which is coherently negative in PtR but assumed positive in Qth, since, following Dirac, authors (also coherently) propose $E = -mc^2$, for $m > 0$. On the other hand, notice that Dirac equations (3.6) or (3.7) result the same if all variables change sign (considering that the *velocity* $\mathbf{v}$ of the particle remains unchanged, in modulus and direction):

$$\begin{cases} \overline{p}_0 = -p_0 \\ \overline{e} = -e \\ \overline{p}_j = \overline{M} v_j = -M v_j = -p_j \quad \text{for } M = \frac{m}{\sqrt{1-\alpha_m^2}} \\ \overline{m} = -m \end{cases}$$

This simply means that *"the fundamental wave equation of the relativistic theory of the electron"* perfectly agrees with Pseudotachyonic Relativity.

Finally, what is the relationship between an antiparticle $\overline{\mathbf{P}}$ and a co-particle $\widehat{\mathbf{P}}$ of the same 'straight' particle $\mathbf{P}$? One should think that $\overline{\mathbf{P}}$ and $\widehat{\mathbf{P}}$ are the same. But yet this isn't so. $\overline{\mathbf{P}}$ (with positive mass) moves forwards in time; according to Feynman, it behaves like $\mathbf{P}$ (also with positive mass) moving backwards in time. The co-particle $\widehat{\mathbf{P}}$ (with negative mass) moves itself backwards in time. If we call "prime-antiparticle' the one corresponding to the 'original' Dirac negative-energy electron, represented by $\mathbf{P}^{\bullet}$, we may summarize the state of all four particles in S in the following table:

$$\mathbf{P}\left\{\begin{array}{l} m+ \\ \\ E+ \\ \\ t+ \end{array}\right. \quad \mathbf{P}^{\bullet}\left\{\begin{array}{l} m+ \\ \\ E- \\ \\ t+ \end{array}\right. \quad \overline{\mathbf{P}}\left\{\begin{array}{l} m+ \\ \\ E+ \\ \\ t- \end{array}\right. \quad \widehat{\mathbf{P}}\left\{\begin{array}{l} m- \\ \\ E- \\ \\ t-, \end{array}\right.$$

which means that $\widehat{\mathbf{P}}$ *is the antiparticle of the co-particle of* $\overline{\mathbf{P}}$.

Of course, with such a confusion, these vital issues raise: does all this picture make sense? Is anybody wrong about it? And, if so, who is it? If not – and this is apparently the case –, how can we understand the existence of two kinds of 'antimatter'? That's exactly what we'll try to find out, and once again – if my hypothesis are true – we'll discover that Nature is a wonderful surprise box.

3.2 Positive and Negative Masses

First of all, one must remark that the *sign* for the mass m has no relevance in the Klein-Gordon equation,

$$\left[\frac{1}{c^2}\frac{\partial^2}{\partial t^2} - \nabla^2 + \frac{m^2c^2}{\hbar^2}\right]\psi = 0, \tag{3.8}$$

which results directly from the relativistic invariant $E^2/c^2 - p^2 = m^2c^2$, by simply replacing E and p by their correspondent operators [9] derived from the wave function $\psi(\mathbf{r}, t) = e^{i/\hbar(\mathrm{p}\cdot\mathrm{r}-\mathrm{Et})}$,

$$E = -\frac{\hbar}{i}\frac{\partial}{\partial t} \quad \Rightarrow E^2 = -\hbar^2\frac{\partial^2}{\partial t^2} \quad \text{and} \quad p = \frac{\hbar}{i}\nabla \quad \Rightarrow p^2 = -\hbar^2\nabla^2.$$

It's important to realize that, in fact, the relativistic invariant $E^2/c^2 - p^2 = m^2c^2$ actually admits positive as negative values for **all** the three variables: energy, momentum and **mass**.

The equation (3.8) applies as well to photons or antiphotons, in the massless case, resulting in the classical wave equation:

$$\left[\frac{1}{c^2}\frac{\partial^2}{\partial t^2} - \nabla^2\right]\psi = 0.$$

Dirac was looking for a linear equation, in which only the first derivatives of the wave function appears. A suggestive way of deducing his equation – and also a striking argument in favour of *negative masses* – is presented in [19]. It's worthy to examine it, reintroducing the constant c and making some adjustments. It begins by expressing the relativistic energy/momentum invariant as a product of two almost similar members by means of basis variables γ_0, γ_1, γ_2 and γ_3:

$$\begin{aligned}&(E/c)^2 - p_x^2 - p_y^2 - p_z^2 - (mc)^2 = \\ &\quad = (\gamma_0 E/c + \gamma_1 p_x + \gamma_2 p_y + \gamma_3 p_z - mc) \times (\gamma_0 E/c + \gamma_1 p_x + \gamma_2 p_y + \gamma_3 p_z + mc)\,.\end{aligned} \tag{3.9}$$

Remark that, since there are both plus and minus signs for m, positive and negative masses give exactly the same result. Now, *"expanding the product and collecting terms, we find that this is a valid equality if and only if the four variables γ_j satisfy the relations*

$$\gamma_0^2 = \mathrm{I} \qquad \gamma_1^2 = \gamma_2^2 = \gamma_3^2 = -\mathrm{I} \qquad \gamma_i\,\gamma_j = -\gamma_j\,\gamma_i \tag{3.10}$$

for all $i,\ j = 0, 1, 2, 3$ with $i \neq j$. (...)

*Focusing on the factor with **positive mass** [The bold is mine], this gives the condition*

$$\gamma_0 E/c + \gamma_1 p_x + \gamma_2 p_y + \gamma_3 p_z - mc = 0. \tag{3.11}$$

Making the usual quantization substitutions for E and p, dividing through by i and $\hbar$, and applying the resulting expression as an operator on a wave function ψ, Dirac arrived at the equation

$$\left[\gamma_0 \frac{1}{c}\frac{\partial}{\partial t} - \gamma_1 \frac{\partial}{\partial x} - \gamma_2 \frac{\partial}{\partial y} - \gamma_3 \frac{\partial}{\partial z} - \mathrm{I}\frac{\mathrm{mc}}{\mathrm{i}\hbar}\right]\psi = 0 \tag{3.12}$$

for a free particle of mass m. (...) He found that the γ_i variables can be represented by 4×4 matrices with complex elements, with the understanding that the symbols I and 0 in equation (3.12) represent the identity matrix and the null matrix respectively. For example, the following matrices satisfy all the requirements":

$$\gamma_0 = \begin{pmatrix} 0 & \mathrm{I} \\ \mathrm{I} & 0 \end{pmatrix} \quad \text{and } \gamma_i = \begin{pmatrix} 0 & \sigma_i \\ -\sigma_i & 0 \end{pmatrix} \text{ for } i = 1, 2, 3,$$

where, like in Dirac equation (3.5), I and 0 are 2×2 matrices and the σ_i are the spin Pauli matrices:

$$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad \sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \quad \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}.$$

It is well known that the square of all three Pauli matrices is the unity matrix

$$\sigma_x\,\sigma_x = \sigma_z\,\sigma_z = \sigma_z\,\sigma_z = \mathrm{I};$$

and that the cross products are anticommutative:

$$\begin{aligned} \sigma_x\,\sigma_y &= i\sigma_z & \sigma_y\,\sigma_x &= -i\sigma_z \\ \sigma_y\,\sigma_z &= i\sigma_x & \sigma_z\,\sigma_y &= -i\sigma_x \\ \sigma_z\,\sigma_x &= i\sigma_y & \sigma_x\,\sigma_z &= -i\sigma_y. \end{aligned}$$

From these remarkable properties we obtain the properties (3.10) concerning the four γ_i.

As we have seen, Dirac equation establishes that the spinor ψ has four components:

$$\psi = \begin{pmatrix} \psi_1 \\ \psi_2 \\ \psi_3 \\ \psi_4 \end{pmatrix}.$$

"Thus, in a sense, we must consider four distinct versions of the particle", states the author of [19]. He refers to particle/antiparticle states together with the two possible states for spin, but he's also very close to another fundamental truth; anyway, he continues by remarking that – and this is also true – the elements of the ψ matrices are not all independent.

To see this and, above all, to establish some interesting new results, we may express – as he does – the vector ψ as a two-dimensional vector of two dimensional vectors ϕ_a and ϕ_b [19]:

$$\psi = \begin{pmatrix} \phi_a \\ \phi_b \end{pmatrix} \quad \text{where } \phi_a = \begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix}, \; \phi_b = \begin{pmatrix} \psi_3 \\ \psi_4 \end{pmatrix}. \tag{3.13}$$

So, *"we can write Dirac wave equation explicitly as*

$$\left[\begin{pmatrix} 0 & \mathrm{I} \\ \mathrm{I} & 0 \end{pmatrix} \frac{\partial}{\partial t} - c \sum_{i=1}^{3} \begin{pmatrix} 0 & \sigma_i \\ -\sigma_i & 0 \end{pmatrix} \frac{\partial}{\partial x^i} \right] \begin{pmatrix} \phi_a \\ \phi_b \end{pmatrix} = \frac{mc^2}{i\hbar} \begin{pmatrix} \phi_a \\ \phi_b \end{pmatrix}. \tag{3.14}$$

Carrying out the matrix multiplications, this represents the following two equations

$$\frac{\partial \phi_b}{\partial t} - c \left(\sigma_x \frac{\partial \phi_b}{\partial x} + \sigma_y \frac{\partial \phi_b}{\partial y} + \sigma_z \frac{\partial \phi_b}{\partial z} \right) = \frac{mc^2}{i\hbar} \phi_a \tag{3.15}$$

$$\frac{\partial \phi_a}{\partial t} + c \left(\sigma_x \frac{\partial \phi_a}{\partial x} + \sigma_y \frac{\partial \phi_a}{\partial y} + \sigma_z \frac{\partial \phi_a}{\partial z} \right) = \frac{mc^2}{i\hbar} \phi_b. \tag{3.16}$$

As a check, by substituting the expression for ϕ_b from the second equation into the first, (3.15), and simplifying by making use of the properties of the spin matrices, we can verify that the two-dimensional vector ϕ_a satisfies the Klein-Gordon equation"

$$\frac{\partial^2 \phi_a}{\partial t^2} - c^2 \left(\frac{\partial^2 \phi_a}{\partial x^2} + \frac{\partial^2 \phi_a}{\partial y^2} + \frac{\partial^2 \phi_a}{\partial z^2} \right) = -\frac{m^2 c^4}{\hbar^2} \phi_a \,, \tag{3.17}$$

and identically for ϕ_b.

As noted before, *"these equations also show that the four components of ψ are not independent, because given any solution ϕ_a of (3.17) we can compute ϕ_b using (3.16). These wave functions will then automatically satisfy (3.15) as well. Therefore, either ϕ_a by itself or ϕ_b by itself is sufficient to determine the complete wave function ψ for a given basis. Also, comparing equations (3.15) and (3.16), we see that ϕ_a and ϕ_a are symmetrical except that the signs of the Pauli spin matrices are reversed. Thus a particle described by Dirac equation has just two possible intrinsic states relative to a given basis, corresponding to the left-handed and right-handed spin states of the particle for that basis".*

The point now is that all this reasoning and set of conclusions remain valid for negative masses, as the author himself implicitly admits. In fact, let us focus now on the factor with **negative mass** (i.e., $-m$) of equation (3.9). There's no reason to discard it, and so the condition (3.9) becomes

$$\gamma_0 E/c + \gamma_1 p_x + \gamma_2 p_y + \gamma_3 p_z + mc = 0, \tag{3.18}$$

similar to (3.11) except for the sign of mc. Alternatively, as in [19], we may arrive at this same equation by introducing the matrix

$$\gamma_5 = i\gamma_0\gamma_1\gamma_2\gamma_3 \quad \Rightarrow \quad \gamma_5 = \begin{pmatrix} -\mathrm{I} & 0 \\ 0 & \mathrm{I} \end{pmatrix}. \tag{3.19}$$

Remark that $\gamma_5^2 = \mathrm{I}$ and also that γ_5 anticommutes with all γ_i:

$$\begin{cases} \gamma_5\gamma_0 = -\gamma_0\gamma_5 = \begin{pmatrix} 0 & -\mathrm{I} \\ \mathrm{I} & 0 \end{pmatrix} \\ \gamma_5\gamma_i = -\gamma_i\gamma_5 = \begin{pmatrix} 0 & -\sigma_i \\ -\sigma_i & 0 \end{pmatrix} \quad \text{for } i = 1, 2, 3. \end{cases}$$

It follows that

$$\gamma_5\left[\gamma_0\tfrac{\partial}{\partial t}-c\textstyle\sum_{i=1}^{3}\gamma_i\tfrac{\partial}{\partial x_i}-I\tfrac{mc^2}{i\hbar}\right]\psi=$$

$$\left[-\gamma_0\tfrac{\partial}{\partial t}+c\textstyle\sum_{i=1}^{3}\gamma_i\tfrac{\partial}{\partial x_i}-I\tfrac{mc^2}{i\hbar}\right]\gamma_5\psi=0$$

"Therefore, if ψ is a solution of Dirac equation (3.12), it follows that $\gamma_5\psi$ is also a solution, the solution of the "negative mass" version of Dirac equation, i.e.,

$$\left[\gamma_0\frac{\partial}{\partial t}-c\sum_{i=1}^{3}\gamma_i\frac{\partial}{\partial x_i}+I\frac{mc^2}{i\hbar}\right]\gamma_5\psi=0 \tag{3.20}$$

Recall that this corresponds to the other factor of the equation $E^2-p^2c^2-m^2c^4=0$, so solutions of this equation are, strictly speaking, equally valid solutions of the Klein-Gordon equation from which we began".

We may summarize this conclusion (making $\psi^{(+)}$ a solution for a positive mass and $\psi^{(-)}$ for the correspondent negative one) as:

$$\begin{cases}\psi^{(-)}=\gamma_5\psi^{(+)}=\begin{pmatrix}-\mathrm{I} & 0\\ 0 & \mathrm{I}\end{pmatrix}\begin{pmatrix}\phi_a\\ \phi_b\end{pmatrix}=\begin{pmatrix}-\phi_a\\ \phi_b\end{pmatrix}\\ \text{and, inversely, } \psi^{(+)}=\gamma_5\psi^{(-)}.\end{cases} \tag{3.21}$$

Further, remark that applying these equations in (3.15) and (3.16), the result for $\psi^{(-)}$ (making $m^*=-m$) is

$$\frac{\partial\phi_b}{\partial t}-\sum_{i=1}^{3}\sigma_i\frac{\partial\phi_b}{\partial x_i}=\frac{mc^2}{i\hbar}\left(-\phi_a\right)=\frac{m^*c^2}{i\hbar}\phi_a$$

$$\frac{\partial\phi_a}{\partial t}+\sum_{i=1}^{3}\sigma_i\frac{\partial\phi_a}{\partial x_i}=\frac{m^*c^2}{i\hbar}\phi_b.$$

So we recover exactly the same equations (3.15) and (3.16).

This simply means that we may permute equations (3.11) and (3.18); this is, we must consider both equations valid either for positive or negative values assigned to the mass m; in one case or the other, the relationship between solutions for $m>0$ and $m<0$ is given by (3.21). As a conclusion, all the equations from (3.1) on are valid either for positive or negative masses.

Finally, one must note that, comparing equations (3.12) and (3.20), there is a reversion of velocity: $p=(-m)(-v)$; this, somehow, corresponds to time reversion.

4 THE FOUR ASPECTS OF MATTER

4.1 The Mass-energy Signature

The generalization of Dirac equation $E=-mc^2$ implies the following equivalence

$$E=\pm\, mc^2 \quad\Leftrightarrow\quad E=\pm\,\mathbf{p}\cdot\hat{\mathbf{v}}, \tag{4.1}$$

the last equation applying as well to massless particles, such as photons.

At this point, to systematize the theory – in fact, to a better comprehension of the subject –, it's useful to introduce the concept of *mass-energy signature* as the ordered pair $(\pm\pm)$ displaying respectively the signs for mass and energy of a particle. We'll say that the standard equation for positive mass m and positive energy E, signature $(++)$, is (3.12), written as

$$\left[\gamma_0 \frac{\partial}{\partial t} - c\sum_{i=1}^{3} \gamma_i \frac{\partial}{\partial x_i} - \mathrm{I}\frac{\mathrm{mc}^2}{\mathrm{i}\hbar}\right]\psi^{(++)} = 0 \quad \text{with} \quad \psi^{(++)} = \begin{pmatrix} \phi_a \\ \phi_b \end{pmatrix}, \tag{4.2}$$

while the standard equation for negative mass $-m$ and positive energy E, signature $(-+)$, is (3.20):

$$\left| \begin{array}{l} \left[\gamma_0 \frac{\partial}{\partial t} - c\sum_{i=1}^{3} \gamma_i \frac{\partial}{\partial x_i} + \mathrm{I}\frac{\mathrm{mc}^2}{\mathrm{i}\hbar}\right]\psi^{(-+)} = 0 \\ \text{with } \psi^{(-+)} = \gamma_5\psi^{(++)} = \begin{pmatrix} -\phi_a \\ \phi_b \end{pmatrix}. \end{array} \right. \tag{4.3}$$

Remark that the expression between right brackets is the same in both equations, except for the sign of the m factor [corresponding to $-m$ in (3.9)].

If we multiply (4.2) by $-\gamma_5$, the solution may be seen as the standard equation for positive mass m and negative energy $-E$, signature $(+-)$:

$$-\gamma_5\left[\gamma_0 \frac{\partial}{\partial t} - c\sum_{i=1}^{3} \gamma_i \frac{\partial}{\partial x_i} - I\frac{mc^2}{i\hbar}\right]\psi^{(++)} =$$

$$\left[-\gamma_0 \frac{\partial}{\partial t} + c\sum_{i=1}^{3} \gamma_i \frac{\partial}{\partial x_i} - I\frac{mc^2}{i\hbar}\right](-\gamma_5)\,\psi^{(++)} = 0,$$

which means that

$$\left| \begin{array}{l} \left[-\gamma_0 \frac{\partial}{\partial t} + c\sum_{i=1}^{3} \gamma_i \frac{\partial}{\partial x_i} - \mathrm{I}\frac{\mathrm{mc}^2}{\mathrm{i}\hbar}\right]\psi^{(+-)} = 0 \\ \text{with } \psi^{(+-)} = -\gamma_5\psi^{(++)} = -\psi^{(-+)} = \begin{pmatrix} \phi_a \\ -\phi_b \end{pmatrix}. \end{array} \right. \tag{4.4}$$

To really understand this, one must notice that the expression between right brackets is the same as (4.2) for $E < 0$ with $p < 0$ (which, once again, because of the equality $E = -p\hat{v}$, implies a velocity reversion for mass-energy signature with opposite signs). One may also notice that, in fact, this expression corresponds exactly to the one for the signature $(-+)$. This means that, in a way, these two signatures are equivalent; in fact, they are the pseudotachyonic transformation one from the other. The same is valid for the signatures $(--)$ and $(++)$

Finally, we'll obtain from (4.3) the standard equation for negative mass $-m$ and negative energy E, signature $(--)$, multiplying it by $-\gamma_5$:

$$-\gamma_5\left[\gamma_0 \frac{\partial}{\partial t} - c\sum_{i=1}^{3} \gamma_i \frac{\partial}{\partial x_i} + \mathrm{I}\frac{\mathrm{mc}^2}{\mathrm{i}\hbar}\right]\psi^{(-+)} =$$

$$\left[-\gamma_0 \frac{\partial}{\partial t} + c\sum_{i=1}^{3} \gamma_i \frac{\partial}{\partial x_i} + I\frac{mc^2}{i\hbar}\right]\underbrace{(-\gamma_5)\,\psi^{(-+)}}_{-\gamma_5^2\psi^{(++)}} = 0$$

or

$$\left| \begin{array}{l} \left[-\gamma_0 \frac{\partial}{\partial t} + c\sum_{i=1}^{3} \gamma_i \frac{\partial}{\partial x_i} + \mathrm{I}\frac{\mathrm{mc}^2}{\mathrm{i}\hbar}\right]\psi^{(--)} = 0 \\ \text{with } \psi^{(--)} = -\gamma_5^2\psi^{(++)} = -\psi^{(++)} = \begin{pmatrix} -\phi_a \\ -\phi_b \end{pmatrix}. \end{array} \right. \tag{4.5}$$

It becomes clear now that Dirac equation describes what we may call "the four aspects of the electron". In general, we may talk about **the four aspects of matter**, each one of them matching a mass-energy signature. So, in 'our' coordinate frame:

- *Straight matter* matches the signature $(++)$;
- *Prime-antimatter*, generalizing Dirac negative-energy electrons, matches the signature $(+-)$;
- *Co-matter* matches the signature $(--)$;
- *Co-prime-antimatter*, the pseudotachyonic transformation of prime-antimatter, matches the signature $(-+)$.

We must keep in mind that Dirac anti-electron (the positron) isn't the *prime-positron* but the particle corresponding to a complex conjugate solution; following the correspondence $\psi \leftrightarrow \bar{\psi}$ we'll write

$$\mathbf{e}^{(m\,;\,-E\,;\,-e)} \leftrightarrow \mathbf{e}^{(m\,;\,E\,;\,+e)}. \qquad (4.6)$$

This means that the negative-energy electron behaves like a positive-energy electron with positive charge. But then we may apply the same reasoning and equations (3.6) and (3.7) to a co-electron; the result is

$$\mathbf{e}^{(-m\,;\,-E\,;\,+e)} \leftrightarrow \mathbf{e}^{(-m\,;\,E\,;\,-e)}. \qquad (4.7)$$

So, the negative-energy co-electron behaves like a positive-energy electron with negative mass; that's why, from a classical point of view, it reacts negatively to Coulomb forces, for instance.

Finally, still following [19], *"since for any given solution ψ, the wave functions $I\psi$ and $\gamma_5\psi$ are also solutions, and since the Dirac equation is linear, any linear combination of solutions is also a solution. Therefore, if we define the matrices $\mathrm{P_L} = (\mathrm{I} - \gamma_5)/2$ and $\mathrm{P_R} = (\mathrm{I} + \gamma_5)/2$, we know that $\mathrm{P_L}\psi$ and $\mathrm{P_R}\psi$ are both solutions, which we will call ψ_L and ψ_R respectively:*

$$\mathrm{P_L} = \begin{pmatrix} \mathrm{I} & 0 \\ 0 & 0 \end{pmatrix} \qquad \mathrm{P_R} = \begin{pmatrix} 0 & 0 \\ 0 & \mathrm{I} \end{pmatrix} \qquad (4.8)$$

Therefore these projection operators resolve the full wave function for a given particle into two parts, namely $\psi = \psi_L + \psi_R$, where the non-zero parts of ψ_L and ψ_R are just the two-dimensional vectors ϕ_a and ϕ_b discussed previously, i.e.,

$$\psi_\mathrm{L} = \begin{pmatrix} \psi_1 \\ \psi_2 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} \phi_a \\ 0 \end{pmatrix} \qquad \psi_\mathrm{R} = \begin{pmatrix} 0 \\ 0 \\ \psi_3 \\ \psi_4 \end{pmatrix} = \begin{pmatrix} 0 \\ \phi_b \end{pmatrix}. \qquad (4.9)$$

As explained previously, ϕ_a and ϕ_a are the same except for having opposite intrinsic spin". As pointed out in subsection 3.2, for ϕ_a and ϕ_b, these components ψ_L and ψ_R correspond to the *left-handed* and *right-handed spin states* of the particle for a given basis. One must bear in mind that the spin isn't but a relativistic effect; as a matter of fact, if the particle is immobile in the reference frame, the sum with Pauli matrices vanish and Dirac equation becomes:

$$\left[\gamma_0 \frac{\partial}{\partial t} - I \frac{mc^2}{i\hbar}\right] = \psi.$$

This is equivalent to, replacing the operator $i\hbar\frac{\partial}{\partial t}$ by E,

$$\begin{pmatrix} 0 & \mathrm{I} \\ \mathrm{I} & 0 \end{pmatrix} E \begin{pmatrix} \phi_a \\ \phi_b \end{pmatrix} = \frac{mc^2}{i\hbar} \begin{pmatrix} \phi_a \\ \phi_b \end{pmatrix},$$

which yields two equations:

$$\begin{cases} -mc^2\phi_a + E\phi_b = 0 \\ E\phi_a - mc^2\phi_b = 0; \end{cases}$$

from the first equation, it comes $\phi_a = \frac{mc^2}{E}\phi_b$; identically, from the second equation, $\phi_b = \frac{mc^2}{E}\phi_a$; and, from both, $E^2 = m^2c^4$ or finally:

$$\begin{cases} \phi_a = \phi_b & \text{for the signatures } (++) \text{ and } (--) \\ \phi_a = -\phi_b & \text{for the signatures } (+-) \text{ and } (-+) \,. \end{cases}$$

According to the standard equations (4.2) to (4.5), we may conclude that:

$$\begin{cases} \psi_{\rm L}^{(++)} = \psi_{\rm L} & \text{and} \quad \psi_{\rm R}^{(++)} = \psi_{\rm R} \\ \psi_{\rm L}^{(-+)} = -\psi_{\rm L} & \text{and} \quad \psi_{\rm R}^{(-+)} = \psi_{\rm R} \\ \psi_{\rm L}^{(+-)} = \psi_{\rm L} & \text{and} \quad \psi_{\rm R}^{(+-)} = -\psi_{\rm R} \\ \psi_{\rm L}^{(--)} = -\psi_{\rm L} & \text{and} \quad \psi_{\rm R}^{(--)} = -\psi_{\rm R} \end{cases} \tag{4.10}$$

5 ALIGNMENT AND ANTI-ALIGNMENT WITH TIME

5.1 Signatures and Lorentz Transformations

As we saw in the previous section, we must incorporate negative masses in Dirac equation and Quantum theory as well as the generalized Einstein equation $E = \pm mc^2$ in Special Relativity (including PtR):

$$\begin{cases} E = mc^2 & \Leftrightarrow \quad E = \mathbf{p}\cdot\hat{\mathbf{v}} & \text{for } (++) \text{ or } (--) \\ E = -mc^2 & \Leftrightarrow \quad E = -\mathbf{p}\cdot\hat{\mathbf{v}} & \text{for } (-+) \text{ or } (+-)\,, \end{cases} \tag{5.1}$$

both theories assuming all the four mass-energy signatures. But where does this last equation come from, in terms of the theory of Relativity itself?

It's a good question. In search for an answer, we must try to understand what these signatures mean, the nature of their mutual and cross relationship. Consider a certain type of what we generically call a 'particle'. Surely, as proposed by PtR, $(++)$ and $(--)$ correspond to a single *'entity'* evaluated in two mutually pseudotachyonic frames; the same is valid for $(+-)$ and $(-+)$. We'll say that each of these correspondences depend on what we may call the ***c* factor**, this is pseudotachyonic transformations. But apparently not the correspondence *matter* $\leftrightarrow$ *prime-antimatter* and therefore not the correspondence *matter* $\leftrightarrow$ *antimatter*.

In fact, the relationship between $(+-)$ and $(++)$ in the same sense of temporal flux cannot match any possible Lorentz transformation, in which energy and time are conjugated variables, transformed in the same way (i.e., both positively or negatively). We therefore have two basic symmetric 'entities', one corresponding to matter, the other to Dirac (prime-)antimatter. But how are they physically related? Why are they symmetric? Or do they also correspond, after all, to a single entity? According to the generalized Dirac theory, the four members of the spinor ψ seem to represent four distinct versions (four possible aspects) of the electron. This is an indication that the four signatures should represent a single entity. In fact, I advocate that this is so. Let's see how and why.

Is it possible to find a way to Relativity, by itself, substantiate these correspondences? This would establish Dirac's negative-energy states as relativistic transformations of coordinates, fully justifying the existence of a single 'entity' in the root of all its four aspects. For a long time, I tried to achieve this goal but all the attempts to develop a matching Lorentz transformation failed... till recently. Finally, I think I've succeeded. I found the solution through a metaphor of the problem that we'll see in a while. Meanwhile, we have to stick to *negative-energy particles* as the real *antiparticles* (and positive-energy particles – conjugate solutions – as just a way of interpret them, nothing else); it is surely simpler because positive-energy as negative-energy electrons are those directly described by Dirac equation. And then we'll need to examine more closely the possibility of temporal reversion on bradyonic transformations, this is *antibradyonic transformations*.

5.2 Time, Energy and Mass

To visualize the new conception these ideas bring, we'll have recourse to a metaphor. Imagine an *archeparticle* $\mathbf{P}^{\circ}$ as an horizontal triangle, turned to the right or to the left, black on the upside (► or ◄) and white on the downside ($\triangleright$ or $\triangleleft$). This triangle is placed between two parallel planes and itself parallel to them; its *image* in each plane result from opposite orthogonal projections, this is:

- the black upside of the triangle in the down plane (say, plane A);
- the white downside of the triangle in the up plane (plane B).

Imagine now that the triangle is pointing to the right (►) and that the plane A is moving from left to right. This moving plane physically corresponds to an ordinary bradyonic frame S and the sense of its movement to the sense of the temporal flux t^+. The projection of ► in A points in the sense of this temporal flux. But if the archeparticle/triangle is pointing to the left (◄), its projection in A points in the direction opposite to temporal flux. In this metaphor, the images of ► and ◄ in A represent respectively the straight particle $\mathbf{P}$ and the prime-antiparticle $\mathbf{P}^{\bullet}$; note that, in this picture, we assign the black color to *positive mass*, as well as the triangle turned in the sense of the planes movement to *positive energy* and opposite to it to *negative energy*.

$$E^+ : \quad ► \quad t^+ \rightarrow ;$$
$$E^- : \quad ◄ \quad t^+ \rightarrow .$$

This is coherent with the following. Consider now the projection of the triangles in the upside plane B, which is moving in the opposite sense to A, this is, from right to left. It represents a pseudotachyonic frame S^*. The projected side of the triangle is now the white one and it appears: 1) pointing in the opposite direction of the temporal flux in the first case; 2) in the same direction in the second case. We may symbolize this by

$$E^{*-} : \quad \triangleright \quad t^{*-} \leftarrow ;$$
$$E^{*+} : \quad \triangleleft \quad t^{*-} \leftarrow .$$

We see that this images correspond to the pseudotachyonic transformation of ► and ◄; the white surface represents *negative mass* and the position of the triangle negative energy in the first case, positive energy in the second.

In this way we may understand that the relationship between planes A and B correspond to pseudotachyonic transformations; but also that the link between Dirac's positive and negative-energy particles (i.e., particles and prime-antiparticles) lies on the double orientation an archeparticle may assume in its 'projection', with respect to time: in the same sense or in opposition to it.

Of course, because of the principle of equivalence, we may turn the triangle/arche-particle upside-down, turning the white face to A and the black face to B. All the previous situations apply as well, resulting four possibilities for whatever reference frame: *the four aspects of an archeparticle*. We may say, somewhat philosophically, that the singular aspect of a particle in whatever reference frame results from a 'projection' of its archeparticle in space-time, from a certain 'point of view'. Note, once again, that in relation to time there are only two possibilities concerning an archeparticle 'projection': in the same direction or in opposition to it.

In this metaphoric approach to the problem, we see that a 'co-particle' isn't the reflected image of a 'particle' in a mirror (as I thought once) but instead the 'opposite' side of the same entity, in the same 'position', that produces the 'particle' image. But we also conclude that the sign for *energy* (the triangle's orientation) may be seen as the alignment $\left(E^+\right)$ or the anti-alignment $\left(E^-\right)$ of the archeparticle's aspect with time flow.

This is worthy of reflection. The movement of each plane represents a 'time arrow', a very strong characteristic of nature in a macroscopic scale; but then, how can it be an anti-alignment with time? Well, on one hand, contrary to classical theories, PtR implies the coexistence of two time flows in the Universe. On the other hand, the possibility itself of *two alignments with time in each reference frame* is a requirement for the relationship between mutually pseudotachyonic frames to exist. Shouldn't the *anti-time nature* manifest in each frame and this relationship would be impossible; we should never 'see' anything else than 'straight' matter and, above all, because of the fundamental part negative energies play in fields mechanics (as proposed in subsection 2.5), the World simply wouldn't work.

So, formally we'll state that:

> The sign for the energy of a particle (massive or not), in whatever reference frame, relates to the *alignment* $(+)$ or the *anti-alignment* $(-)$ of this particle towards positive time flow in that frame.

In fact, this is quite plausible in view of the direct relationship between energy and frequency established by Planck and generalized by de Broglie: $E = h\nu$; positive or negative energies correspond respectively to positive or negative frequencies. Ultimately, then, the alignment or anti-alignment of the particle towards time flow arise from the ***h* factor**.

After all, this relevant conclusion should be evident from the beginning, because of the close relationship between the 4-vectors $\mathbf{x}^\mu = (ict,\, \mathbf{x})$ and $(iE/c,\, \mathbf{p})$: $\mathbf{p}$ relates to the alignment (the movement) of the particle in the three-dimensional space; E relates to the alignment (the movement) of the particle in the temporal dimension. But in the end this is absolutely coherent with the fact that energy changes sign under pseudotachyonic transformation (which relates two coordinate frames where time flows in opposite direction):

- $E^- \to E^{*+}$: the particle becomes aligned with time in the frame S^*; from the point of view of this coordinate frame, the particle is no longer moving 'against' time;
- $E^+ \to E^{*-}$: the particle becomes anti-aligned with time in the frame S^*.

These and other speculations suggest the need to define the **alignment factor** $\diamond = \diamond_m \cdot \diamond_E$ ($\diamond = 1$ for $+$ and $\diamond = -1$ for $-$), allowing to write the system of equations (5.1) in a single line:

$$E = \diamond\, mc^2 \quad \Leftrightarrow \quad E = \diamond\, \mathbf{p}\cdot\hat{\mathbf{v}} \qquad (5.2)$$

Note that *this factor* $\diamond$ *is invariant under Lorentz bradyonic or pseudotachyonic transformations* because both members of the mass-energy signature maintain or change sign jointly. As we'll see in the next subsection, *it is anti-invariant under Lorentz antibradyonic (or antipseudotachyonic) transformations*.

In parallel, we should update the equations (20), (22) and (23) as

$$\begin{aligned} &E = -\diamond\, \tfrac{\beta}{\alpha} E_0^*; \quad m = -\diamond\, \tfrac{\beta}{\alpha} m_0^*; \\ &E_k = \left(1 - \frac{1}{\sqrt{1-(\hat{v}/c)^2}}\right) \diamond\, m_0^* c^2\,. \end{aligned} \qquad (5.3)$$

Remark that, instead of characterizing a particle for its mass-energy signature, we may do it through its *alignment-energy signature*, displaying the correspondent sign of $\diamond$ and the sign of energy, $[\pm\pm]$, which has the advantage of applying as well to massless particle, such as photons. In 'our' coordinate frame:

- *Straight matter* matches the signature $[++]$;
- *Prime-antimatter* matches $[--]$;
- *Co-matter* matches $[+-]$;
- *Co-prime-antimatter* matches $[-+]$.

In short, the alignment factor relates to the orientation of the archeparticle projection with respect to time flux in a certain reference frame and is the 'responsible' for Dirac's double solution concerning the energy of the electron.

Finally, what about *photons*? We'll represent massless particles as arrows, without surface. For these particles, there is no difference in 'black or white' projections on the up or the down plane (this is S or S^*); but there's still an alignment or an anti-alignment with time:

$$\begin{aligned} &\left(\gamma \text{ or } \widehat{\overline{\gamma}}\right) \quad E^+: \quad \longrightarrow \quad t^+ \to\,; \\ &\left(\widehat{\gamma} \text{ or } \overline{\gamma}\right) \quad E^-: \quad \longleftarrow \quad t^+ \to\,. \end{aligned}$$

5.3 Antibradyonic Lorentz Transformations

The final point is to understand how we must interpret the reversion of time in the frame S. It simply corresponds to reverse time in the usual Lorentz transformation. It's quite useful to introduce a frame $S^\bullet$ moving backwords in time, symmetrically to its paraframe S'; this is, $t^\bullet = -t'$ and $x_j^\bullet = x_j'$ for $j = 1, 2, 3$. We'll call this an *antibradyonic Lorentz transformation* and, according to (2.4), we'll have:

$$\begin{cases} \mathbf{A}^{\bullet 0} = \frac{i\beta \mathbf{A}^1 - \mathbf{A}^0}{\sqrt{1-\beta^2}} \\ \mathbf{A}^{\bullet 1} = \frac{\mathbf{A}^1 + i\beta \mathbf{A}^0}{\sqrt{1-\beta^2}} \\ \mathbf{A}^{\bullet 2} = \mathbf{A}^2 \\ \mathbf{A}^{\bullet 3} = \mathbf{A}^3. \end{cases} \tag{5.4}$$

Remark that the inverse transformations result identical to this one, permuting tensors:

$$\begin{cases} \mathbf{A}^0 = \frac{i\beta \mathbf{A}^1 - \mathbf{A}^{\bullet 0}}{\sqrt{1-\beta^2}} \\ \mathbf{A}^1 = \frac{\mathbf{A}^{\bullet 1} + i\beta \mathbf{A}^{\bullet 0}}{\sqrt{1-\beta^2}} \\ \mathbf{A}^2 = \mathbf{A}^{\bullet 2} \\ \mathbf{A}^3 = \mathbf{A}^{\bullet 3}. \end{cases} \tag{5.5}$$

So, in the case of the space-time 4-vector, we'll get

$$\begin{cases} t^\bullet = \frac{vx/c^2 - t}{\sqrt{1-\beta^2}} \\ x^\bullet = \frac{x - vt}{\sqrt{1-\beta^2}} \\ y^\bullet = y \\ z^\bullet = z, \end{cases} \tag{5.6}$$

It's easy to see that, since $dx_j^\bullet = dx_j'$ $(j = 1, 2, 3)$ and $dt^\bullet = -dt'$, we obtain for the composition of velocities:

$$\mathbf{u}^\bullet = -\mathbf{u}' \quad \Rightarrow \quad \begin{cases} u_x^\bullet = \frac{u_x - v}{1 - vu_x/c^2} \\ u_y^\bullet = \frac{u_y\sqrt{1-\beta^2}}{1 - vu_x/c^2} \\ u_z^\bullet = \frac{u_y\sqrt{1-\beta^2}}{1 - vu_x/c^2}; \end{cases} \tag{5.7}$$

and for acceleration:

$$\mathbf{a}^\bullet = \frac{d}{dt^\bullet}\mathbf{u}^\bullet = -\frac{d}{dt'}(-\mathbf{u}') \quad \Rightarrow \quad \mathbf{a}^\bullet = \mathbf{a}'. \tag{5.8}$$

All this makes us think about pseudotachyonic transformations but it is not exactly the same. In particular, the examination of this problem according to the Galilean transformation shows that the mass should be kept positive (for instance, an uniformly accelerated motion becomes uniformly slowed by the reversion of time; the related force becomes opposite to movement, assuming that the mass is invariant). This agrees with the metaphorical approach of the previous section but implicates that we must conceive the energy-momentum 4-vector in $S^\bullet$ as

$$\left(iE^\bullet/c,\, -p_x^\bullet,\, -p_y^\bullet,\, -p_z^\bullet\right) \tag{5.9}$$

this is, in tensorial trasnformations:

$$\begin{cases} \mathbf{A}^{\bullet 0} = iE^\bullet/c \\ \mathbf{A}^{\bullet j} = -p_j^\bullet \qquad \text{for } j = 1, 2, 3. \end{cases} \tag{5.10}$$

In a way, this corresponds to the operator (3.3) – this is the negative Hamiltonian – used by Dirac to obtain the invariant (3.4), while (3.2) – the positive Hamiltonian - corresponds to the 4-vector $(iE/c,\, p_x,\, p_y,\, p_z)$.

Applying now the antibradyonic rules (5.4) and (5.5) to this energy-momentum 4-vector, we get the following transformation tables:

$$\begin{cases} E^\bullet = \frac{vp_x - E}{\sqrt{1-v^2/c^2}} \\ p_x^\bullet = \frac{Ev/c^2 - p_x}{\sqrt{1-v^2/c^2}} \\ p_y^\bullet = -p_y \\ p_z^\bullet = -p_z; \end{cases} \quad \text{and inversely} \quad \begin{cases} E = -\frac{vp_x^\bullet + E^\bullet}{\sqrt{1-v^2/c^2}} \\ p_x = -\frac{vE^\bullet/c^2 + p_x^\bullet}{\sqrt{1-v^2/c^2}} \\ p_y = -p_y^\bullet \\ p_z = -p_z^\bullet; \end{cases} \tag{5.11}$$

These inverse transformations may not seem identical – and this puzzled me for a while! – but one must remember that there's a reversion of sign for $\mathbf{p}_j^\bullet$ and $\mathbf{p}_j$. As a matter of fact, we'll have

$$\begin{cases} E^{\bullet} = -E' \\ \mathbf{p}^{\bullet} = -\mathbf{p}'. \end{cases} \tag{5.12}$$

Remark that, as one would expect,

$$\begin{cases} \mathbf{p}' = m'\mathbf{u}' = m'\mathbf{u}^{\bullet} \\ \mathbf{p}^{\bullet} = m^{\bullet}\mathbf{u}^{\bullet} = -\mathbf{p}' \end{cases} \quad \Rightarrow \quad m^{\bullet} = m' = \frac{m_0}{\sqrt{1-v^2/c^2}} \tag{5.13}$$

and, since $E^{\bullet} = -E' = -m'c^2$, it comes that

$$E^{\bullet} = -m^{\bullet}c^2. \tag{5.14}$$

This is precisely Dirac's identity for his negative-energy solution, clearly suggesting that this solution refers to an antibradyonic Lorentz transformation, with the understanding that it implicates time reversion. Its quite relevant to note that time doesn't directly figure in Dirac equation. Furthermore, we may prove, in a similar way we do to pseudotachyonic transformations [see Apendix A], that *electric charge* is anti-invariant under antibradyonic transformation: $e^{\bullet} = -e$. This means that every *archeparticle* shows off in S and in $S^{\bullet}$ with opposite charges. And this, in a way, meets the modern interpretation of antiparticles as particles going backwards in time: an antiparticle $\mathbf{P}^{\bullet}$, with positive mass $m^{\bullet}$, negative energy $E^{\bullet}$ and charge q in an antibradyonic frame $S^{\bullet}$ appears in our frame $S \equiv S'$ as a particle $\mathbf{P}$ moving symmetrically, with the same mass, positive energy $E = -E^{\bullet}$ and opposite charge $-q$.

5.4 Quadrivalent Special Relativity

We have established the four aspects that an archeparticle may assume as the result of four variations of the basic Lorentz transformations. We'll say that all these variations – bradyonic, antibradyonic, pseudotachyonic and anti-pseudotachyonic transformations – are embraced by a **Quadrivalent Special Relativity** (QSR).

There is still much to say. But, for now, a simple final note must be done regarding the fundamental concept of "proper frame" (or "rest frame"), which has a clear meaning in classical Relativity for a material particle. An immediate consequence of QSR is that **there is no "proper frame" of a particle in an absolute sense**, this is, concerning its archeparticle, but only in a relative sense, concerning one of its four aspects. As a matter of fact, the notion of "rest" along with the assignment of $\pm$ signs to time, energy, mass, etc. relies on measurements of the space-time reality, this is, on frames of coordinates. The point is that the archeparticle is independent of frames of coordinates and therefore only its 'projection' in one of them reveals it existence, in one of its four aspects; we may talk then of the proper frame of the 'particle', as the one where the particle is at rest.

This is important in the sense that it eliminates a certain privilege we would be inclined to attribute to positive energy or mass; and this would violate the equivalence principle. At least, one should attribute a privilege to the alignment of positive energy with time, in the sense that it corresponds to the *emission* (and not the absorption) of mediators by the particle as the source of a field of forces – which seems to be more 'natural'. But, in the end, this simply comes to strengthen the fundamental relevance of the *time arrow*.

6 CONCLUSION

We conclude that Special Relativity admits four variations for Lorentz transformations: two for $|v| < c$, this is, *bradyonic* and *antibradyonic* transformations; and two for $|v| > c$: *pseudotachyonic* and *anti-pseudotachyonic* transformations.

The first case is the usual transformation. The third (PtR) implicates a reversion of time and, so, the fundamental co-existence of two time flows in the Universe; it shows that tachyons must be detected as co-particles, with negative mass and negative energy. The second case corresponds to an inversion of time but lower than light velocity; it concerns Dirac prime-antiparticles, with positive masses but negative energy. Finally, the fourth is a combination of the second and the third; the detected particles have negative masses and positive energy.

From an epistemological point of view, QSR brings a (potential) unifying simplification of particles and force fields. For instance, here lies the answer to the question: why does every particle have its own antiparticle? In fact, the crucial consequence of this theory is that **all four homologous particles aren't but aspects of a single 'entity'**, we may call their **'archeparticle'**. Similar conclusions may be achieved to massless particles such as photons. Furthermore, it becomes clear that positive and negative elementary charges have the same magnitude because they result from a single 'archecharge'. A basis for the study of force fields based on positive and negative energies is outlined and the identity of inertial and gravitational masses is strongly induced.

COMPETING INTERESTS

Author has declared that no competing interests exist.

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APPENDIX

A Appendix: Electric charge in pseudotachyonic transformations

I'll just transcribe here the arguments presented in [2]. Since, with regard to time, a pseudotachyonic frame S^* and its paraframe S' behave symmetrically, we must admit that the element of *proper time* of a particle (moving in relation to S with velocity $\mathbf{u}$), is in pseudotachyonic transformations *anti-invariant*: $d\tau = -dt^*\sqrt{1-(u^*/c)^2}$. Remark that this agrees with the second equation in (2.15).
Now, the 4-vector [7]

$$c^a = \rho_0 \tfrac{dx^a}{d\tau} \quad (a = 0, 1, 2, 3)$$

is the density of current charge, ρ_0 being the *proper electric charge density* in a certain point, measured by a local observer, and $\frac{dx^a}{d\tau}$ the 4-vector velocity (in relation to τ). So, for bradyonic transformations, in which $d\tau = dt\sqrt{1-(u/c)^2}$,

$$c^a = \rho_0 \frac{dx^a}{dt}\frac{dt}{d\tau} = \frac{\rho_0}{\sqrt{1-(u/c)^2}}\frac{dx^a}{dt}.$$

Defining $\rho = \rho_0/\sqrt{1-(u/c)^2}$ as the *charge density* measured in the frame S, in which the charge has a velocity u at the instant t, we may write

$$c^a = \rho \tfrac{dx^a}{dt} = \rho u^a \quad \text{or} \quad \begin{cases} c^0 = ic\rho \\ c^1 = \rho u_x \\ c^2 = \rho u_y \\ c^3 = \rho u_z. \end{cases}$$

The 4-vector c^a also transforms according to the bradyonic and pseudotachyonic laws for a generic contravariant 4-vector $\mathbf{A}^a$, (2.5) and (2.7) respectively. The result for the *time component* of c^a is

$$c^{*0} = \frac{ic^1 - \beta c^0}{\alpha} = ic\rho\frac{u_x/c - \beta}{\alpha};$$

but $c^{*0} = ic\rho^*$, and, as a consequence,

$$\rho^* = \rho\frac{u_x/c - \beta}{\alpha}. \tag{A.1}$$

It follows that, inversely to what occurs in bradyonic transformations, in pseudotachyonic transformations the charge density ρ is *anti-invariant*. In fact, we may deduce from velocity transformation – equations (2.14) – the relation

$$\frac{\sqrt{1-(u/c)^2}}{\sqrt{1-(u^*/c)^2}} = \frac{\beta - u_x/c}{\alpha}, \tag{A.2}$$

which means, according to (A.1), that

$$\rho^*\sqrt{1-(u^*/c)^2} = -\rho\sqrt{1-(u/c)^2} = -\rho_0. \tag{A.3}$$

Now, due to the inversion of time, the element of quadridimensional volume – which is invariant in bradyonic transformations – is anti-invariant in the pseudotachyonic case $[dx^*dy^*dz^*dt^* = -dx\,dy\,dz\,dt]$; but then, reminding that

$$\frac{dt^*}{dt} = \frac{u_x/c - \beta}{\alpha}$$

and applying the equality (A.2), the element of tridimensional volume, in ordinary space, transforms according to the equation

$$\frac{dv}{\sqrt{1-(u/c)^2}} = \frac{dv^*}{\sqrt{1-(u^*/c)^2}}. \tag{A.4}$$

On the other hand, since

$$\begin{cases} e = \rho\, dv \\ e^* = \rho^*\, dv^* \end{cases}$$

are the values obtained respectively in S and in S^* for the *electric charge* of a material particle, it results that

$$e = -e^*. \tag{A.5}$$

In short, the electric charge (invariant in bradyonic transformations) is *anti-invariant* in pseudotachyonic transformations. So, *the electric charge of a co-particle* $\widehat{\mathbf{P}}$ *must be* opposite *to that of its homologous* $\mathbf{P}$.

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