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Cubic Fuzzy Positive Implicative Extension Ideals in BCK-Algebras

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Abstract

An interval-valued fuzzy set denotes a fuzzy set whose membership function is many -valued and form an interval in the member ship scale. The notion of cubic sets as an extension of fuzzy set in which not only a membership degree is given but also a non-membership degree is involved. In this present study, we introduce the concept of cubic fuzzy positive implicative ideals, cubic fuzzy positive implicative extension ideals in BCK-algebras and investigate some of its properties.

Keywords: BCK-algebras; fuzzy positive implicative ideal; fuzzy positive implicative extension ideals.

1 Introduction and Preliminaries

The notion of interval-valued fuzzy sets was first introduced by Zadeh [1] as an extension of fuzzy sets. An interval-valued fuzzy set denotes a fuzzy set whose membership function is many -valued and form an interval in the member ship scale. This idea gives the simplest method to capture the imprecision of the membership grade for a fuzzy set. On the other hand, Jun et al. [2] introduced the notion of cubic sets as an extension of fuzzy set in which not only a membership degree is given but also a non-membership degree is involved. In this paper we introduce the concept of cubic fuzzy positive implicative ideal, cubic fuzzy ideal extensions of BCK-algebras and investigate some of its properties.

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Satyanarayana and Durga Prasad [3] A BCK-algebra is a non-empty set X with a binary operation $*$ and a constant 0 satisfying the following axioms:

(BCK-1) $(x * y) * (x * z) \leq (z * y)$.

(BCK-2) $x * (x * y) \leq y$.

(BCK-3) $x \leq x$.

(BCK-4) $x \leq y\ , y \leq x \Rightarrow x = y$.

(BCK-5) $0 \leq x$, where $x \leq y$ is defined by $x * y = 0$.

A BCK-algebra can be partially ordered by $x \leq y \Leftrightarrow x * y = 0$ this ordering is called BCK-ordering. In any BCK-algebra X the following hold:

$(P1)$ $x * 0 = x$,

$(P2)$ $x * y \leq x$,

$(P3)$ $(x * y) * z = (x * z) * y$,

$(P4)$ $(x * z) * (y * z) \leq x * y$,

$(P5)$ $x * \big(x * (x * y)\big) = x * y$,

$(P6)$ $x \leq y \Longrightarrow x * z \leq y * z$ and $z * y \leq z * x$ for every $x , y , z \in X$.

Definition 1.1. [3] A Subset I of a BCK-algebra $(X\ ,\ * , 0)$ is called an ideal of X , for any $x , y \in X$

(B1) $0 \in I$,

(B2) $x * y$ and $y \in I \Rightarrow x \in I$.

Definition 1.2. [3] An ideal I of a BCK-algebra $(X,\ * , 0)$ is called closed if $0 * x \in I$, for all $x \in I$.

Definition 1.3. [4,5] A non-empty Subset I of a BCK-algebra $(X,\ * , 0)$ is said to be a positive implicative ideal if it satisfies,

(B3) $0 \in I$,

(B4) $(x * y) * z \in I$ and $y * z \in I \Rightarrow x * z \in I$, for all $x, y, z \in X$.

We now review some fuzzy logic concepts. A fuzzy set in X is a function $\mu : X \to [0,1]$.

For fuzzy sets μ and λ of X and $s, t \in [0,1]$. The sets $U\ (\mu\ ; t) = \{x \in X :\ \mu(x) \geq t\}$ is called upper t-level cut of μ and $L(\lambda; s) = \{x \epsilon\ X : \lambda(x) \leq s\}$ is called lower s-level cut of λ. The fuzzy set μ in X is called fuzzy sub algebra of X, if $\mu(x * y) \geq \min\{\mu(x), \mu(y)\}$,, for all $x, y \in X$.

By the interval number D we mean an interval $[\ a^{-}, a^{+}]$ where $0 \leq a^{-} \leq a^{+} \leq 1$

For interval numbers $D_1 = [a_1^{-}, b_1^{+}]$, $D_2 = [a_2^{-}, b_2^{+}]$.

We define

$\min(D_1, D_2) = D_1 \cap D_2 = \min([a_1^-, b_1^+], [a_2^-, b_2^+]) = [\min\{a_1^-, a_2^-\}, \min\{b_1^+, b_2^+\}]$

$\max(D_1, D_2) = D_1 \cup D_2 = \max([a_1^-, b_1^+], [a_2^-, b_2^+]) = [\max\{a_1^-, a_2^-\}, \max\{b_1^+, b_2^+\}]$

$D_1 + D_2 = [a_1^- + a_2^- - a_1^-.a_2^-,\ b_1^+ + b_2^+ - b_1^+.b_2^+]$ and put

$D_1 \le D_2 \Leftrightarrow a_1^- \le a_2^-$ and $b_1^+ \le b_2^+$

$D_1 = D_2 \Leftrightarrow a_1^- = a_2^-$ and $b_1^+ = b_2^+$

$D_1 < D_2 \Leftrightarrow D_1 \le D_2$ and $D_1 \ne D_2$

$mD = m[a_1^-, b_1^+] = [ma_1^-, mb_1^+]$, where $0 \le m \le 1$.

Let L be a given nonempty set [6]. An interval valued fuzzy set B on L is defined by

$B = \{x, [\mu_B^-(x), \mu_B^+(x)] : x \in L\}$, where $\mu_B^-(x)$ and $\mu_B^+(x)$ are fuzzy sets of L such that $\mu_B^-(x) \le \mu_B^+(x)$ for all $x \in L$. Let $\tilde{\mu}_B(x) = [\mu_B^-(x), \mu_B^+(x)]$, then B = {(x,$\tilde{\mu}_B(x)$): $x \in L$} where $\tilde{\mu}_B : L \to D[0,1]$

Biswas [7] described a method to find maximum and minimum between two set of intervals

Definition 1.4: [7] Consider two set of intervals $D_1, D_2 \in D[0,1]$. If $D_1 = [a_1^-, a_1^+]$ then $\text{rmin}(D_1, D_2) = [\min(a_1^-, a_2^-), \min(a_1^+, a_2^+)]$ which is denoted by $D_1 \wedge^r D_2$. Thus if $D_i = [a_i{}^-, a_i{}^+] \in D[0,1]$ for $1 \le i \le n$ then we define $r\sup_i(D_i) = [\sup_i(a_i^-), \sup_i(a_i^+)]$ that is $\vee_i^r D_i = [\vee_i a_i^-, \vee_i a_i^+]$. Now we call $D_1 \ge D_2$ iff $a_1^- \ge a_2^-$ and $a_1^+ \ge a_2^+$. Similarly the relations $D_1 \le D_2$ and $D_1 = D_2$ are defined.

Based on fuzzy sets, Jun et al. [2] introduced the notion of cubic sets and investigated several properties.

Definition 1.5: [2] Let X be a non-empty set. A cubic set A in X is a Structure $A = \{(x, \tilde{\mu}_A(x), \lambda_A(x))\,; x \in$ X which is briefly denoted by $A=\mu A,\lambda A$ where $\mu A=\mu A-,\mu A+$ is an interval valued fuzzy set in X and λA is fuzzy set in X.

Definition 1.6: [8] A cubic set $A = (X, \tilde{\mu}_A, \lambda_A)$ in X is a cubic ideal of X, if it satisfies

(CI 1) $\tilde{\mu}_A(0) \ge \tilde{\mu}_A(x)$ and $\lambda_A(0) \le \lambda_A(x)$

(CI 2) $\tilde{\mu}_A(x) \ge \text{rmin}\{\tilde{\mu}_A(x * y), \tilde{\mu}_A(x)\}$

(CI 3) $\lambda_A(x) \le \max\{\lambda_A(x * y), \lambda_A(y)\}$ for all $x, y \in X$.

2 Main Results

Definition 2.1. A cubic fuzzy set $A = (X, \tilde{\mu}_A, \lambda_A)$ in BCK- algebra X is called a cubic fuzzy BCK-positive implicative ideal if it satisfies

(C BCK -PI-1) $\tilde{\mu}_A(0) \ge \tilde{\mu}_A(x)$ and $\lambda_A(0) \le \lambda_A(x)$

(C BCK-PI-2) $\tilde{\mu}_A(x * z) \ge \text{rmin}\{\tilde{\mu}_A((x * y) * z), \tilde{\mu}_A(y * z)\}$

(C BCK-PI-3) $\lambda_A(x * z) \le \max\{\lambda_A((x * y) * z), \lambda_A(y * z)\}$ for all $x, y, z \in X$.

Theorem 2.2. Every cubic fuzzy BCK positive implicative ideal is a cubic ideal of X but the converse is not true.

Proof: Let cubic fuzzy set $A = (X, \tilde{\mu}_A, \lambda_A)$ is a cubic fuzzy BCK-positive implicative ideal. By definition 2.1, we have

$\tilde{\mu}_A(0) \geq \tilde{\mu}_A(x)$ and $\lambda_A(0) \leq \lambda_A(x)$

$\tilde{\mu}_A(x * z) \geq$ rmin $\{\tilde{\mu}_A((x * y) * z), \tilde{\mu}_A(y * z)\}$ and

$\lambda_A(x * z) \leq \max\{\lambda_A((x * y) * z), \lambda_A(y * z)\}$ for all $x, y, z \in X$.

Putting $z = 0$ we get

$\tilde{\mu}_A(x * 0) \geq$ rmin $\{\tilde{\mu}_A((x * y) * 0), \tilde{\mu}_A(y * 0)\}$

$\tilde{\mu}_A(x) \geq$ rmin $\{\tilde{\mu}_A(x * y), \tilde{\mu}_A(y)\}$ and

$\lambda_A(x * 0) \leq \max\{\lambda_A((x * y) * 0), \lambda_A(y * 0)\}$

$\Rightarrow \lambda_A(x) \leq \max\{\lambda_A(x * y), \lambda_A(y)\}$.

Hence cubic fuzzy BCK- positive implicative ideal $A = (\tilde{\mu}_A, \lambda_A)$ is a cubic fuzzy ideal of X and the converse of this is not true it can be shown through the following example

Consider the BCK-algebra X ={0,1,2,3,4} with the following canyley table:

*	0	1	2	3	4
0	0	0	0	0	0
1	1	0	1	0	0
2	2	2	0	0	0
3	3	3	3	0	0
4	4	3	4	1	0

Define a cubic fuzzy set $A = (\tilde{\mu}_A, \lambda_A)$ in X by $\tilde{\mu}_A(0) = [0.6, 0.7]$,

$\tilde{\mu}_A(1) = \tilde{\mu}_A(2) = \tilde{\mu}_A(3) = [0.1, 0.2]$ and $\tilde{\mu}_A(4) = [0.1, 0.3]$, $\lambda_A(0) = \lambda_A(1) = 0.1$, $\lambda_A(2) = \lambda_A(3) = 0.3$, $\lambda_A(4) =0.4$.

It is easy to verify that it is a cubic ideal but it is not a cubic fuzzy BCK-positive implicative ideal of X because $\tilde{\mu}_A(4 * 3) = \tilde{\mu}_A(1) = [0.1, 0.2$, and $\min\{\tilde{\mu}_A((4 * 1) * 3), \tilde{\mu}_A(1 * 3)\} =$ min $\{\tilde{\mu}_A(0), \tilde{\mu}_A(0)\} = \tilde{\mu}_A(0) = [0.6, 0.7]$ Clearly [0.1, 0.2] < [0.6,0.7]

Corollary 2.3: Let $A = (X, \tilde{\mu}_A, \lambda_A)$ be a cubic fuzzy BCK-positive implicative ideal of X. If $x \leq y$ in X, then $\tilde{\mu}_A(x) \geq \tilde{\mu}_A(y)$ and $\lambda_A(x) \leq \lambda_A(y)$, that is, $\tilde{\mu}_A$ is an order-reversing and λ_A is an order-preserving.

Theorem 2.4: A cubic fuzzy set $A = (\tilde{\mu}_A, \lambda_A)$ is a cubic ideal of a BCK- algebra X. Then A is a cubic fuzzy positive implicative ideal of X if and only if it satisfies the inequalities

$\tilde{\mu}_A((x * z) * (y * z)) \geq \tilde{\mu}_A((x * y) * z)$ and $\lambda_A((x * z) * (y * z)) \leq \lambda_A((x * y) * z)$ for all $x, y, z \in X$.

Proof: Assume that $A = (X, \tilde{\mu}_A, \lambda_A)$ is a cubic fuzzy positive implicative ideal of X. By theorem 2.3.

$A = (X, \tilde{\mu}_A, \lambda_A)$ is a cubic fuzzy ideal of X. Let $x, y, z \in X$ and $a = x * (y * z)$ and $b = (x * y)$. Since

$$\left((x * (y * z)) * ((x * y) * z)\right) \leq ((y * (y * z)) * z).$$

By corollary 2.3, we have

$$\tilde{\mu}_A\left((x * (y * z)) * ((x * y) * z)\right) \geq \tilde{\mu}_A((y * (y * z)) * z)$$

and

$$\lambda_A\left((x * (y * z)) * ((x * y) * z)\right) \leq \lambda_A((y * (y * z)) * z).$$

Then $\tilde{\mu}_A\big((a * b) * z\big) = \tilde{\mu}_A\left(\left((x * (y * z)) * (x * y)\right) * z\right)$

$$\geq \tilde{\mu}_A\left((y * (y * z)) * z\right) \text{ (by BCK-1and Corollary 2.3)}$$

$$= \tilde{\mu}_A\big((y * z) * (y * z)\big)$$

$$= \tilde{\mu}_A(0).$$

By (C BCK -PI-1) we have $\tilde{\mu}_A\big((a * b) * z\big) = \tilde{\mu}_A(0)$,

and

$$\lambda_A\big((a * b) * z\big) = \lambda_A\left(\left((x * (y * z)) * (x * y)\right) * z\right)$$

$$\leq \lambda_A\left((y * (y * z)) * z\right) \text{ (by BCK-1and Corollary 2.3)}$$

$$= \lambda_A\big((y * z) * (y * z)\big)$$

$$= \lambda_A(0).$$

By (C BCK -PI-1) we have $\lambda_A\big((a * b) * z\big) = \lambda_A(0)$.

Using (P3), (CPI2) and (CPI3) we obtain

$$\tilde{\mu}_A\big((x * z) * (y * z)\big) = \tilde{\mu}_A\left((x * (y * z)) * z\right)$$

$$= \tilde{\mu}_A(a * z)$$

$$\geq \text{rmin}\left\{\tilde{\mu}_A\big((a * b) * z\big), \tilde{\mu}_A(b * z)\right\}$$

$$= \text{rmin}\{\tilde{\mu}_A(0), \tilde{\mu}_A(b * z)\}$$

$$= \tilde{\mu}_A(b * z)$$

$$= \tilde{\mu}_A\big((x * y) * z\big)$$

and

$$\lambda_A\big((x*z)*(y*z)\big) = \lambda_A\Big(\big(x*(y*z)\big)*z\Big)$$

$$= \lambda_A(a*z)$$

$$\leq \max\{\lambda_A\big((a*b)*z\big), \lambda_A(b*z)\}$$

$$= \max\{\lambda_A(0), \lambda_A(b*z)\}$$

$$= \lambda_A(b*z)$$

$$= \lambda_A\big((x*y)*z\big)$$

Thus $\tilde{\mu}_A\big((x*z)*(y*z)\big) \geq \tilde{\mu}_A\big((x*y)*z\big)$ and $\lambda_A\big((x*z)*(y*z)\big) \leq \lambda_A\big((x*y)*z\big)$ for all $x \in X$.

Conversely, suppose that $A = (X, \tilde{\mu}_A, \lambda_A)$ is a cubic ideal of X satisfies the inequalities

$$\tilde{\mu}_A\big((x*z)*(y*z)\big) \geq \tilde{\mu}_A\big((x*y)*z\big) \text{ and}$$

$$\lambda_A\big((x*z)*(y*z)\big) \leq \lambda_A\big((x*y)*z\big) \text{ for all } x, y, z \in X.$$

Using (CI-2) and (CI-3) for all $x, y, z \in X$. We obtain

$$\tilde{\mu}_A(x*z) \geq \text{rmin}\{\tilde{\mu}_A\big((x*z)*(y*z)\big), \tilde{\mu}_A(y*z)\}$$

$$\geq \text{rmin}\{\tilde{\mu}_A\big((x*y)*z\big), \tilde{\mu}_A(y*z)\}$$

and

$$\lambda_A(x*z) \leq \max\{\lambda_A\big((x*z)*(y*z)\big), \lambda_A(y*z)\}$$

$$\leq \max\{\big((x*y)*z\big), \lambda_A(y*z)\}$$

Thus $A = (X, \tilde{\mu}_A, \lambda_A)$ is a cubic fuzzy positive implicative ideal of X.

Theorem 2.5. If $A = (\tilde{\mu}_A, \lambda_A)$ is a cubic fuzzy positive implicative ideal of X, then the non-empty upper $[s_1, s_2]$ level cut $U(\tilde{\mu}_A; [s_1, s_2])$ and the non-empty lower t-level cut $L(\lambda_A; t)$ are positive implicative ideals of X.

Proof: Let $x \in U(\tilde{\mu}_A, [s_1, s_2]) \Rightarrow \tilde{\mu}_A(x) \geq [s_1, s_2]$

$$\Rightarrow [s_1, s_2] \leq \tilde{\mu}_A(x) \leq \tilde{\mu}_A(0)$$

$$\Rightarrow \tilde{\mu}_A(0) \geq [s_1, s_2]$$

$$\Rightarrow 0 \in U(\tilde{\mu}_A; [s_1, s_2])$$

Let $x, y, z \in X$ be such that $\big((x*y)*z\big) \in U(\tilde{\mu}_A; [s_1, s_2])$, $y*z \in U(\tilde{\mu}_A; [s_1, s_2])$

$$\tilde{\mu}_A\big((x*y)*z\big) \geq [s_1, s_2] \text{ and } \tilde{\mu}_A(y*z) \geq [s_1, s_2].$$

Since

$$\tilde{\mu}_A\ (x * z) \geq \text{rmin}\left\{\tilde{\mu}_A\big((x * y) * z\big), \tilde{\mu}_A(y * z)\right\}$$

$$\geq \text{rmin}\left\{\,[s_1, s_2\,], [s_1, s_2]\right\}$$

$$= \{\min\,[s_1, s_1], \min\,[s_2, s_2]\ \}$$

$$= [s_1, s_2\,]$$

Therefore $x * z \in U\,(\tilde{\mu}_A\ ;[\ s_1, s_2\,])$

Hence $U\,(\tilde{\mu}_A\ ;[\ s_1, s_2\,])$ is a positive implicative ideal of X.

Let $x \in L\,(\lambda_A; t) \implies \lambda_A(x) \leq t$

$\implies \lambda_A(0) \leq \lambda_A(x) \leq t$

$\implies 0 \in L(\lambda_A; t)$.

Further more if

$(x * y) * z \in L(\lambda_A; t)$ and $y * z \in L(\lambda_A; t)$

then

$\lambda_A\big((x * y) * z\big) \leq t$ and $\lambda_A(y * z) \leq t$.

Since

$$\lambda_A(x * z) \leq \max\left\{\,\lambda_A\big((x * y) * z\big),\ \lambda_A(y * z)\right\} \leq \max\,\{t, t\} = t,$$

$$\implies \lambda_A\,(x * z) \leq t \implies x * z \in L(\lambda_A; t).$$

Hence $L(\lambda_A; t)$ is a positive implicative ideal of X.

Definition 2.6: Fuzzy ideal extension: [9] Let μ be a fuzzy subset of a BCK-algebra X and $a \in X$. Then the fuzzy subset $< \mu, a >: X \rightarrow [0,1]$ defined by $< \mu, a > (x) = \mu(x * a)$ is called extension of μ by a.

Definition 2.7: Cubic fuzzy ideal extension:

Let $(\tilde{\mu}_A, \lambda_A)$ be a cubic fuzzy set in a BCK-algebra X and $a, b \in X$. Then the cubic fuzzy set $< (\tilde{\mu}_A, \lambda_A), (a, b) >$ defined by $< (\tilde{\mu}_A, \lambda_A), (a, b) > = (< \tilde{\mu}_A, a >, < \lambda_A, b >)$ is called the extension of $(\tilde{\mu}_A, \lambda_A)$ by (a, b). If $a = b$ then it is denoted by $< (\tilde{\mu}_A, \lambda_A), a >$.

Theorem 2.8: Let a cubic fuzzy set $A = (\tilde{\mu}_A, \lambda_A)$ be a cubic fuzzy BCK- positive implicative ideal of a BCK-algebra X and $a, b \in X$. Then the extension $< (\tilde{\mu}_A, \lambda_A), (a, b) >$ of $(\tilde{\mu}_A, \lambda_A)$ by $(a, b)$is also a cubic fuzzy BCK-positive implicative ideal of X.

Proof: Suppose that a cubic fuzzy set $A = (\tilde{\mu}_A, \lambda_A)$ be a cubic fuzzy BCK- positive implicative ideal of a BCK-algebra X and $a, b \in X$. Let $x, y, z \in X$. Then we have

$$< \tilde{\mu}_A, a > (0) = \tilde{\mu}_A(0 * a)$$
$$= \tilde{\mu}_A (0)$$
$$\geq \tilde{\mu}_A(x * a)$$
$$=< \tilde{\mu}_A, a > (x) \text{ and}$$

$$< \lambda_A, b > (0) = \lambda_A(0 * b) = \lambda_A(0) \leq \lambda_A(x * b) = < \lambda_A, b > (x)$$

Now $< \tilde{\mu}_A, a > (x * z) = \tilde{\mu}_A\big((x * z) * a\big) = \tilde{\mu}_A\big((x * a) * (z * a)\big)$

$$\geq \text{rmin}\left\{\tilde{\mu}_A\left(\big((x * a) * (y * a)\big) * (z * a)\right), \tilde{\mu}_A\big((y * a) * (z * a)\big)\right\}$$
$$= \text{rmin}\left\{\tilde{\mu}_A\left(\big((x * y) * z\big) * a\right), \tilde{\mu}_A\big((y * z) * a\big)\right\}$$
$$= \text{rmin}\left\{< \tilde{\mu}_{A,} a > \big((x * y) * z\big), < \tilde{\mu}_{A,} a > (y * z)\right\}$$

Similarly $< \lambda_A, b > (x * z) = \lambda_A\big((x * z) * b\big)$

$$= \lambda_A\big((x * b) * (z * b)\big)$$
$$\leq \max\left\{\lambda_A\left(\big((x * b) * (y * b)\big) * (z * b)\right), \lambda_A((y * b) * (z * b)\right\}$$
$$= \max\left\{\lambda_A\left(\big((x * y) * z\big) * b\right), \lambda_A((y * z) * b)\right\}$$
$$= \max\left\{< \lambda_A, b > \big((x * y) * z\big), < \lambda_A, b > (y * z)\right\}.$$

Hence the extension $< (\tilde{\mu}_A,\lambda_A), (a, b) >$ of $(\tilde{\mu}_A,\lambda_A)$ by (a, b) is a cubic fuzzy BCK-positive implicative ideal of X.

Corollary 2.9. Let a cubic fuzzy set $A = (\tilde{\mu}_A,\lambda_A)$ be a cubic fuzzy BCK- positive implicative ideal of a BCK-algebra X and $a \in X$. Then the extension $< (\tilde{\mu}_A,\lambda_A), a >$ of $(\tilde{\mu}_A, \lambda_A)$ by a is also a cubic fuzzy BCK-positive implicative ideal of X.

3 Conclusion

In this paper cubic fuzzy BCK positive implicative ideals in BCK-algebra are introduced. Example is given in support of the definition of cubic fuzzy BCK positive implicative ideals. Some theorems are proved. And also introduced cubic fuzzy ideal extension and proved one theorem. It will help to improve the concept of fuzzy ideals. There are many other aspects which should be explored and studied in the area of fuzzy ideals such as cubic fuzzy implicative ideals and cubic fuzzy commutative ideals.

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Competing Interests

Authors have declared that no competing interests exist.

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