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Applied Artificial Intelligence

An International Journal

ISSN: (Print) (Online) Journal homepage: https://www.tandfonline.com/loi/uaai20

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To cite this article: Abdi Oli & Gerima Tefera (2023) Fuzzy derivations of d-ideals of d-algebras and Cartesian product of Fuzzy derivation of d-ideals of d-algebras, Applied Artificial Intelligence, 37:1, 2157938, DOI: 10.1080/08839514.2022.2157938

To link to this article: https://doi.org/10.1080/08839514.2022.2157938

Published online: 30 Dec 2022.

APPLIED ARTIFICIAL INTELLIGENCE
2023, VOL. 37, NO. 1, e2157938 (108 pages)
https://doi.org/10.1080/08839514.2022.2157938

Fuzzy derivations of d-ideals of d-algebras and Cartesian product of Fuzzy derivation of d-ideals of d-algebras

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ABSTRACT
The concepts of left (right) fuzzy derivations of d-ideals of d-algebra is introduced. The cartesian product of left (right) fuzzy derivations of d-ideals are investigated. Different characterizations of right (left) fuzzy derivation of ideals of d-algebra are discussed.

ARTICLE HISTORY
Received 11 November 2022
Accepted 7 December 2022

1. Introduction

After the introduction of fuzzy set theory by Zadeh (1965), different fuzzification of the concepts of crisp set to fuzzy set become the major results. Jana et al. (2017) introduced t-derivations on a complicated subtraction algebras and Mostefa, Abd-Elnaby, and Yousef (2011) initiated the concept of fuzzy left (right) derivations of Ku-ideal of Ku-algebras. Senapati et al. (2019, 2021) initiated the idea of cubic intuitionistic sub-algebras and closed cubic intuitionistic ideals of B-algebras and cubic intuitionistic structures applied to ideals of BCI-algebra. The notion of fuzzy left (right) derivations of BCC-ideals in BCC-algebras, and the Cartesian product of fuzzy left (right) derivations of BCC-ideals introduced by Jun et al. (1999). Gerima and Fasil (2020) introduced the concept of derivations in a BF-algebra. In addition, a left-right and a right-left derivation of BF_2-algebra, left and right derivation of ideal in BF-algebras were discussed. The notion of d-algebras and d-ideals was introduced by Neggers and Kim (1999). Left (right)-derivation of d-ideal of d-algebra was discussed by Young Hee kim (1018), and Akram and Dar (2005) introduced the idea of fuzzy d-algebras. This concept was extended to structure of Fuzzy dot d-sub-algebras by Gerima Tefera (2020). The concept of P_0 – almost distributive fuzzy lattices with different characterization was introduced by Berhanu et al. (2022) and characterization of homomorphism in implication algebra intiated by Tefera (2022). The concepts of fuzzy derivations of ideals of BF-algebra with different properties discussed by Gerima and Tsige (2022). These mentioned ideas motivated us to introduced the concepts fuzzy derivation in d-algebra as a new concept.

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2. 2.Preliminaries

Definition 2.1. Mostefa, Abd-Elnaby, and Yousef (2011) An Algebra $(X, *, 0)$ of type $(2, 0)$ is called a $BCK-$ algebra if it satisfies the following conditions:

(i) $x * x = 0$
(ii) $0 * x = 0$
(iii) $x * y = 0$ and $y * x = 0$ implies $x = y$ for all $x, y \in X$
(iv) $((x * y) * (x * z)) * (z * y) = 0,$
(v) $((x * (x * y)) * y = 0.$

Definition 2.2. Neggers and Kim (1999) A nonempty set X with a constant 0 and a binary operation * is called a d-algebra, if it satisfies the following axioms:

(i) $x * x = 0$
(ii) $0 * x = 0$
(iii) $x * y = 0$ and $y * x = 0$ implies $x = y$ for all $x, y \in X$

Let S be a non-empty subset of a d-algebra X, then S is called a sub-algebra of X if $x * y \in S$ for all $x, y \in S$ (Gerima Tefera 2020).

Definition 2.3. (Neggers, Jun, and Kim 1999) Let X be a $d-$ algebra and I be a subset of X, then I is called d-ideal of X if it satisfies following conditions:

(a) $0 \in I$
(b) $x * y \in I$ and $y \in I,$ implies $x \in I.$
(c) $x \in I$ and $y \in X$ implies $x * y \in I$, i.e. $IxX \subseteq I$

Definition 2.4. (Zadeh 1965) Let X be a non-empty set. A fuzzy (sub)set μ of the set X is a mapping $\mu : X \to [0, 1]$.

Definition 2.5. (Akram and Dar 2005) A fuzzy set μ in d-algebra X is called a fuzzy sub-algebra of X if it satisfies $\mu(x * y) \geq \min\{\mu(x), \mu(y)\}$, for all $x, y \in X$.

Definition 2.6. (Hee Kim 2018) Let $(X, *, 0)$ be a $d-$ algebra and let $x \wedge y := y * (y * x)$ for all $x, y \in X$. The map$d : X \to X$ is said to be an $(r, l)-$ derivation if $d(x * y) = (x * d(y)) \wedge (d(x) * y)$ for all $x, y \in X$. Similarly, a map $d : X \to X$ said to be an $(l, r)-$ derivation if $d(x * y) = (d(x) * y) \wedge (x * d(y))$ for all$x, y \in X$.

Definition 2.7 (Akram and Dar 2005) A fuzzy set μ in X is called fuzzy $d-$ ideal of X if it satisfies the following inequalities:

(Fd1) $\mu(0) \geq \mu(x)$,

(Fd2) $\mu(x) \geq min\{\mu(x * y), \mu(y)\}$,

(Fd3) $\mu(x * y) \geq min\{\mu(x), \mu(y)\}$ for all x, y $\in$ X.

Definition 2.8. (Jun and Xin 2004) Let $(X, *, 0)$ be $BCC-$ algebra a fuzzy set μ in X is called fuzzy derivation of $BCC-$ ideal of X if it satisfies the following conditions:

(a) $\mu(0) \geq \mu(x) \quad x \in X$
(b) $\mu(d(x * z)) \geq \min\{\mu(d(x * y) * y), \mu(d(y))\}$for all $x, y \in X$

Definition 2.9. (Akram and Dar 2005) Let μ be the fuzzy set of a set X. For a fixed $s \in [0, 1]$, the set $\mu_s = \{x \in X : \mu(x) \geq s\}$ is called an upper level of μ.

A fuzzy subset μ is called fuzzy relation on a set S, if μ is a fuzzy subset $\mu : S\,x\,S \to [0, 1]$ (Akram and Dar 2005) .

Definition 2.10. (Akram and Dar 2005) If μ is a fuzzy relation on a set S and β is fuzzy subset of S, then μ is a fuzzy relation on β if $\mu(x, y) \leq \min\{\beta(x), \beta(y)\} \quad x, y \in S$.

3. Main Results

3.1. Fuzzy Derivatives of D-Ideals of D-Algebra

Definition 3.1.1. Let be X a $d-$ algebra and $d : X \to X$ be a self-map. A fuzzy subset $\mu : X \to [0, 1]$ in X called a fuzzy right derivation of $d-$ ideal of X, if it satisfies the following conditions:

$a.$ $\mu(0) \geq \mu(x) \quad x \in X$.
$b.$ $\mu(d(x)) \geq \min\{\mu(x * d(y)), \mu(d(y))\} \quad x, y \in X$.

Example 3.1.1. Let $X = \{0, 1, 2, 3, 4\}$ be a set and $*$ be defined by the table below:

*	0	1	2	3	4
0	0	0	0	0	0
1	1	0	1	2	1
2	2	2	0	3	0
3	3	3	2	0	3
4	4	4	4	1	0

Table 3.1 Fuzzy right derivation of d-ideal of d-algebra

Then $(X, *, 0)$ is $d-$ algebras.

Now define self-map $d : X \to X$ by $d(x) = \begin{cases} 1, & if\ x = 0, 3 \\ 0, & if\ x = 2,\ 4 \\ 2, & if\ x = 1 \end{cases}$

And define a fuzzy derivation $\mu : d(x) \to [0, 1]$ by $\mu(d(0)) = \mu(d(3)) = \mu(1) = t_1$,$\mu(d(2)) = \mu(d(4)) = \mu(0) = t_0$, $\mu(d(1)) = \mu(2) = t_2$ where $t_0 > t_1 > t_2$ and $t_0, t_1, t_2 \in [0, 1]$.

i. $\mu(0) \geq \mu(x) \quad x \in X$

Since $\mu(0) = \mu(x * x) \geq \min\{\mu(x), \mu(x)\} = \mu(x)$

ii. $\mu(d(x)) \geq \min\{\mu(x * d(y)), \mu(d(y))\}$

Then let $x = 1, y = 1$. So

$\mu(d(1) \geq min\{\mu(1 * d(2))\}, \mu(d(2))\} = min\{\mu(1 * 0)\}, \mu(0)\}$
$= min\{\mu(1), \mu(0)\} = t_1$.

Thus, $\mu(d(1)) \geq \mu(1) = t_1$

Let $x = 2, y = 1$ then

$\mu(d(2)) \geq \min\{\mu(2 * d(1)), \mu(d(1))\}$-
$= \min\{\mu(2 * 2), \mu(2)\} = \min\{\mu(0), \mu(2)\} = t_2$

Thus, $\mu(d(2)) \geq \mu(2) = t_2$

Let $x = 3, y = 1$ then

$\mu(d(3)) \geq \min\{\mu(3 * d(1)), \mu(d(1))\}$-
$= \min\{\mu(3 * 2), \mu(2)\} = \min\{\mu(2), \mu(2)\} = \mu(2) = t_2$

But $\mu(d(3)) = \mu(1) = t_1 > t_2$. Thus, $\mu(d(3)) \geq \mu(2) = t_2$

Let $x = 3, y = 2$ then

$\mu(d(3)) \geq \min\{\mu(3 * d(2)), \mu(d(2))\}$-
$= \min\{\mu(3 * 0), \mu(0)\} = \min\{\mu(3), \mu(0)\} = \mu(3) = t_2$

Note: $\mu(x * d(y)) = 1 - \mu(d(y))$

Let $x = 3, y = 3$ then

$\mu(d(3)) \geq \min\{\mu(3 * d(3)), \mu(d(3))\}$
$= \min\{\mu(3 * 1), \mu(1)\} = \min\{\mu(3), \mu(1)\} = \mu(3) = 1 - \mu(d(3))$

Let $x = 3, y = 4$ then

$\mu(d(3)) \geq \min\{\mu(3 * d(4)), \mu(d(4))\}$
$= \min\{\mu(3 * 0), \mu(0)\} = \min\{\mu(3), \mu(0)\} = \mu(3) = 1 - \mu(d(3))$
Let $x = 2$, $y = 3$ then
$\mu(d(2)) \geq \min\{\mu(2 * d(3)), \mu(d(3))\}$
$= \min\{\mu(2 * 1), \mu(1)\} = \min\{\mu(2), \mu(1)\} = \mu(3) = \mu(2) = t_2$
In any case, $\mu$is fuzzy right derivation of $d-$ ideal of $d-$ algebra of X.

Remark 3.1.1. In the above example **(3.1.1)** μ is **no**t fuzzy left derivation of $d-$ ideal of $d-$ algebra of X.

Definition 3.1.2. Let X be a $d-$ algebra and $d : X \to X$ be a self--map. A fuzzy set $\mu : X \to [0,1]$ in X called a fuzzy left derivation of $d-$ ideal, if it satisfies the following conditions:

a. $\mu(0) \geq \mu(x) \quad x \in X$
b. $\mu(d(x)) \geq \min\{\mu(d(x) * y), \mu(d(y))\}$ for all $x, y \in X$

Example 3.1.2. Let $X = \{0, 1, 2\}$ be a set and $*$ be defined by the table below:

*	0	1	2
0	0	0	0
1	1	0	2
2	2	1	0

Table 3.2 Fuzzy left derivation of d-ideal of d-algebra.
Then $(X, *, 0)$ is $d-$ algebras.

Now define self-map $d : X \to X$by $d(x) = \begin{cases} 0, & if\, x = 0, 1 \\ 1, & if\ x = 2 \end{cases}$

And define a fuzzy derivation $\mu : d(X) \to [0,1]$by $\mu(d(0)) = \mu(d(1)) = \mu(1) = t_0$,$\mu(d(2)) = \mu(1) = t_1$ where $t_0 > t_1 > t_2$ and $t_0, t_1, t_2 \in [0,1]$.

Now by using the definition of fuzzy left derivations of $d-$ ideal of $d-$ algebra X we can show that $\mu : d(X) \to [0,1]$ is fuzzy left derivation of $d-$ ideal of $d-$ algebra X as follows:

i. $\mu(0) \geq \mu(x) \quad x \in X$
Since $\mu(0) = \mu(x * x) \geq \min\{\mu(x), \mu(x)\} = \mu(x)$
ii. $\mu(d(x)) \geq \min\{\mu(d(x) * y), \mu(d(y))\}$

Then let$x = 1,\ y = 2$. So

$$\mu(d(1)) \geq \min\{\mu(d(1) * 2), \mu(d(2))\}$$
$$= \min\{\mu(0 * 2), \mu(1)\} = \min\{\mu(0), \mu(1)\} = \mu(1) = t_1$$

Then let $x = 1,\ y = 1$. So

$$\mu(d(1)) \geq \min\{\mu(d(1) * 1), \mu(d(1))\}$$
$$= \min\{\mu(0 * 1), \mu(0)\} = \min\{\mu(0), \mu(0)\} = \mu(0) = t_0$$

Then let $x = 0,\ y = 1$. So

$$\mu(d(0)) \geq \min\{\mu(d(0) * 1), \mu(d(1))\}$$
$$= \min\{\mu(0 * 1), \mu(0)\} = \min\{\mu(0), \mu(0)\} = \mu(0) = t_0$$

Let $x = 0,\ y = 2$. So

$$\mu(d(0)) \geq \min\{\mu(d(0) * 2), \mu(d(2))\}$$
$$= \min\{\mu(0 * 2), \mu(1)\} = \min\{\mu(0), \mu(1)\} = \mu(1) = t_1$$

Let $x = 2,\ y = 1$. So

$$\mu(d(2)) \geq \min\{\mu(d(2) * 1), \mu(d(1))\}$$
$$= \min\{\mu(1 * 1), \mu(0)\} = \min\{\mu(0), \mu(0)\} = \mu(0) = t_0$$

But $\mu(d(2)) = \mu(1) = t_1 < t_0$. Which is contradiction with $\mu(d(2)) = \mu(1) = t_1 \geq t_0$

Thus, μ is **no**t fuzzy left derivation of $d-$ ideal of $d-$ algebra of X.

Example 3.1.3. Let $X = \{0, a, b, c\}$ be a set with * given by the following table:

*	0	a	b	c
0	0	0	0	0
a	a	0	c	b
b	b	c	0	a
c	c	c	b	0

Table 3.3 Left derivation of d – ideal of d – algebra

Then $(X, *, 0)$ is $d-$ algebras.

Now define self-map $d : X \to X$by $d(x) = \begin{cases} 0, & if\ x = 0, a \\ b, & if\ x = b, c \end{cases}$

And define a fuzzy derivation $\mu : d(X) \to [0, 1]$ by $\mu(d(0)) = \mu(d(a)) = \mu(0) = t_0$,$\mu(d(b)) = \mu(d(b)) = \mu(b) = t_1, \mu(c) = t_2$, where $t_0 > t_1 > t_2$ and $t_0, t_1, t_2 \in [0, 1]$.

i. $\mu(0) \geq \mu(x) \quad x \in X$.

Since $\mu(0) = \mu(x * x) \geq \min\{\mu(x), \mu(x)\} = \mu(x)$
In the same method it is easy to show the remaining part.

Hence $\mu$is fuzzy left derivation of $d-$ ideal of $d-$ algebra of X.

Definition 3.1.3. Let $\mu : X \rightarrow [0,1]$ be fuzzy subset of X and X is $d-$ algebra. Let $\alpha \in [0,1]$, then $\mu_\alpha = \{x \in X/\mu(d(x)) \geq \alpha\}$ is level subset of μ.

Definition 3.1.4. [4] Let μ be fuzzy set of a set X. For a fixed $s \in [0,1]$, the set $\mu_s = \{x \in X/\mu(x) \geq s\}$ is called an upper level of μ.

Theorem 3.1.1. *Let μ be a fuzzy set in X then μ is a fuzzy left derivations of $d-$ ideal of X if and only if it satisfies : For all $\alpha \in [0,1], \mu_\alpha \neq \emptyset$ implies μ_α is $d-$ ideal of X where $\mu_\alpha = \{x \in X/\mu(d(x)) \geq \alpha\}$.*

Proof. Let μbe a fuzzy subset in X

$(\Rightarrow)$: (1). Assume that μbe a fuzzy left derivations $d-$ ideal of X.

Let μ be a fuzzy left derivations of $d-$ ideal of X and $\alpha \in [0,1]$ such that $\mu_\alpha \neq \emptyset$ and for $x, y \in X$with $\mu(d(x)) \neq \emptyset$ and $\mu(d(x)) \geq \alpha$ then $d(x) \in \mu_\alpha$ and $d(y) \in \mu_\alpha$.

$$\mu(d(0)) = \mu(d(x * x)) \geq \min\{\mu(d(x) * (y * x)), \mu(d(y))\} \geq \alpha.$$

$$\Rightarrow \mu(d(0)) \geq \alpha$$

$$\Rightarrow d(0) \in \mu_\alpha$$

Hence, $0 \in \mu_\alpha = \cup(\mu, \alpha)$

(2). $d(x) * y \in \mu_\alpha$ *and* $d(y) \in \mu_\alpha$

Then $\mu(d(x) * y) \geq \alpha$ *and* $\mu(d(y)) \geq \alpha$

$$\Rightarrow \min\{\mu(d(x) * y), \mu(d(y))\} \geq \alpha$$

Since, μ is a fuzzy left derivation of $d-$ ideal of d-algebra X,

$$\mu(d(x)) \geq \min\{\mu(d(x) * y), \mu(d(y))\} \geq \alpha$$

$$\Rightarrow \mu(d(x)) \geq \alpha$$

And hence, $d(x) \in \mu_\alpha$.

(3). Let $d(x) \in \mu_\alpha$ and $y \in X$ we have to show $d(x) * y \in \mu_\alpha$

Now $\mu(d(x) * y) = \mu(d(x * 0) * y) \geq \min\{\mu(d(x) * (y * 0) * y), \mu(d(y))\}$

$$= \min\{\mu(d(x) * (y * y)), \mu(d(y))\}$$

$$= \min\{\mu(d(x) * 0), \mu(d(y))\}$$

$$= \min\{\mu(d(x)), \mu(d(y))\} \geq \alpha$$

Therefore, $\mu(d(x) * y) \geq 0$

$$\Rightarrow d(x) * y \in \mu_\alpha = \cup(\mu, \alpha)$$

And hence, μ_α is a $d-$ ideal of X.

Conversely, assume that μ satisfies $\cup(\mu, \alpha) = \{x \in X/\mu(d(x)) \geq \alpha\}$. Let $x, y \in X$

$$\mu(d(x)) < \min\{\mu(d(x) * y), \mu(d(y))\}$$

By taking $\beta_0 = \frac{1}{2}[\mu(d(x)) + \min\{\mu(d(x) * y), \mu(d(y))\}]$
We have $\beta_0 \in [0, 1]$ and $\mu(d(x)) < \beta_0 < \min\{\mu(d(x) * y), \mu(d(y))\}$
It follows that:

$d(x) * y \in \cup(\mu, \alpha)$and $d(x) \in \cup(\mu, \beta_0)$

It contradicts and therefore μ is a fuzzy left derivations $d-$ ideal of X.

Theorem 3.1.2. *Let μ be a fuzzy set in X, then μ is a fuzzy right derivations of $d-$ ideal of X if and only if it satisfies : For all $\alpha \in [0, 1], \mu_\alpha \neq \emptyset$ implies μ_α is $d-$ ideal of X where $\mu_\alpha = \{x \in X/\mu(d(x)) \geq \alpha\}$.*

Proof. It is similar with the prove of the above theorem **(3.1.1)**. So, we omitted the proof.

Proposition 3.1.1. The intersection of any family of fuzzy derivation of d-ideal of fuzzy left derivations $d-$ ideal of $d-$ algebra $X$is also fuzzy left derivations $d-$ ideal.

Proof. Let $\{\mu_i\}_{i \in I}$ be a family of fuzzy left derivations $d-$ideals of $d-$ algebra$X$, then for any $x, y \in X$.

$$(\cap\mu_i)(d(x)) = \inf(\mu_i(d(x)))$$

$$\geq \inf(\min\{\mu_i(d(x) * y), \mu_i(d(y))\})$$

$$= \min\{\inf(\mu_i(d(x) * y)), \inf(\mu_i(d(y)))\}$$

$$= \min\{(\cap\mu_i)(d(x) * y), (\cap\mu_i)(d(y))\}$$

Lemma 3.1.1. The intersection of any family of fuzzy derivation of right derivations $d -$ ideals of $d -$ algebra X is also fuzzy right derivations $d -$ ideal.

Definition 3.1.5. Let μ and β be fuzzy left derivations subset of a set S, the Cartesian product of μ and β is defined by $(\mu \times \beta)(d(x), d(x)) = \min\{\mu(d(x), \beta(d(x)\}, \quad x, y \in S.$

Definition 3.1.6 . If μ is a fuzzy left derivations on a set S and β is a fuzzy left derivation on β if $\mu(d(x), d(y)) \le \min\{\beta(d(x)), \beta(d(y))\} \quad x, y \in S.$

Definition 3.1.7. Let μ and β be fuzzy left derivations subset of a set S. Then the Cartesian product of μ and β is defined by $(\mu \times \beta)(d(x), d(y)) = \min\{\mu(d(x)), \mu(d(y))\} \quad x, y \in S.$

Theorem 3.1.3. *Let μ and β be fuzzy left derivations $d -$ ideals of $d -$ algebra X, then $\mu \times \beta$ is a fuzzy left derivations $d -$ ideal of $X \times X$.*

Proof. for any $(x, y) \in X \times X,$ we have

$$(\mu \times \beta)(d(0), d(0)) = \min\{\mu(d(0)), \beta(d(0))\}$$

$$= \min\{\mu(0), \beta(0)\}(\because \beta(d(x)) \le \beta(d(0)) = \beta(0))$$

$$\ge \min\{\mu(d(x)), \beta(d(x))\}$$

$$= (\mu \times \beta)(d(x), d(x))$$

Now let $(x_1, x_2), (y_1, y_2) \in X \times X,$ then, $(\mu \times \beta)(d(x_1), d(x_2)) = \min\{\mu(d(x_1), \beta(d(x_2)\}$

$$\ge \min\{\min\{\mu(d(x_1) * \mu(d(y_1)))\}, \min\{\beta(d(x_2) * y_2), \beta(d(y_2))\}\}$$

$$= \min\{\min\{\mu(d(x_1) * y_1), \mu(d(x_2) * y_2)\}, \min\{\mu(d(y_1)), \beta(d(y_2))\}\}$$

$$= \min\{(\mu \times \beta)(d(x_1) * y_1,\ d(x_2)\ * y_2), (\mu \times\ \beta)(d(y_1), d(y_2))\}$$

Hence, $\mu \times \beta$ is a fuzzy left derivation $d -$ ideal of $X \times X$.

Definition 3.1.8. If β is a fuzzy left derivations subset of a set S, the strongest fuzzy relation on S, that is a fuzzy derivation relation on β is μ_β given by $\mu_\beta(d(x,y)) = \min\{\beta(d(x)), \beta(d(y))\} \quad x, y \in S.$

Proposition 3.1.2. For a given fuzzy subset β of $d-$ algebra X, let μ_β be the strongest left fuzzy derivation relation on X. If μ_β is fuzzy left derivation $d-$ ideal of $X x X$, then $\beta(d(x)) \leq \beta(d(0)) = \beta(0)$ for all $x \in X$.

Theorem 3.1.4. *Let β be a fuzzy subset of $d-$ algebra Xand let μ_β be the strongest fuzzy left derivation X, then β is a fuzzy left derivation $d-$ algebra of X if and only if μ_β left derivation $d-$ algebra of $X x X$.*

Proof. Assume that β a fuzzy left derivation $d-$ algebra X, we note from (F_1) that

$$\mu_\beta(0,0) = \min\{\beta(d(0)), \beta(d(0))\}$$

$$= \min\{\beta(0), \beta(0)\}$$

$$\geq \min\{\beta(d(x)), \beta(d(y))\}$$

$$= \mu_\beta(d(x), d(y))$$

Now, for any $(x_1, x_2), (y_1, y_2) \in X x X$ we have from (F_2):

$$\mu_\beta(d(x_1), d(x_2)) = \min\{\beta(d(x_1)), \beta(d(x_2))\}$$

$$\geq \min\{\min\{\beta(d(x_1) * y_1), \beta(d(y_1))\}, \min\{\beta(d(x_2) * y_2), \beta(d(y_2))\}\}$$

$$= \min\{\min\{\beta(d(x_1) * y_1), \beta(d(x_2) * y_2)\}, \min\{\beta(d(y_1)), \beta(d(y_2))\}\}$$

$$= \min\left\{\mu_\beta(d(x_1) * y_1, (d(x_2) * y_2)), \mu_\beta(d(y_1), d(y_2))\right\}$$

Hence, μ_β is fuzzy left derivation $d-$ ideal of $X x X$.
Conversely, for all $(x, y) \in X x X$, we have
$\min\{\beta(0), \beta(0)\} = \mu_\beta(x, y) = \min\{\beta(x), \beta(y)\}$ it follows:
$\beta(0) \geq \beta(x) \quad x \in X$ which prove (F_1).
Let $(x_1, x_2), (y_1, y_2) \in X x X$then

$$\min\{\beta(d(x_1)), \beta(d(x_2))\} = \mu_\beta(d(x_1), d(x_2))$$

$$\geq \min\left\{\mu_\beta(d(x_1) * y_1), \mu_\beta(d(y_1), d(y_2))\right\}$$

$$= \min\left\{\mu_\beta(d(x_1) * y_1, d(x_2) * y_2), \mu_\beta(d(y_1), d(y_2))\right\}$$

$$= \min\{\min\{\beta(d(x_1) * y_1), \beta(d(x_2) * y_2)\}, \min\{\beta(d(y_1)), \beta(d(y_2))\}\}$$

$$= \min\{\min\{\beta(d(x_1) * y_1), \beta(d(y_1))\}, \min\{\beta(d(x_2) * y_2), \beta(d(y_2))\}\}$$

In particular, if we take $x_2 = y_2 = 0$ then, $\beta(d(x_1)) \geq \min\{\beta(d(x_1) * y_1), \beta(d(y_1))\}$
This prove $(FL_2)$and complete the prove.

4. Conclusion

The notion of left (right) fuzzy derivations of d-ideals of d-algebra is introduced. The Cartesian product of left (right) fuzzy derivations of d-ideals is discussed. Strong fuzzy relations with illustrative examples are explained. Different characterizations theorems are proved. As a future work the authors indicate these idea can be extend to contra derivation of fuzzy ideals of d-algebra, Cubic intuitionistic sub algebras on d-algebra.

Disclosure statement

No potential conflict of interest was reported by the authors.

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