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REVERSE ZAGREB AND REVERSE HYPER-ZAGREB INDICES FOR SILICON CARBIDE $Si_2C_3I[r, s]$ AND $Si_2C_3II[r, s]$

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ABSTRACT. Topological indices collect information from the graph of molecule and help to predict properties of underlined molecule. Zagreb indices are among the most studied topological indices due to its applications in chemistry. In this report we compute first and second reversed Zagreb indices and first and second reversed Hyper Zagreb indices for $Si_2C_3I[r, s]$ and $Si_2C_3II[r, s]$. Moreover we also compute first and second reversed Zagreb polynomials and first and second reversed Hyper Zagreb polynomials for $Si_2C_3I[r, s]$ and $Si_2C_3II[r, s]$.

Index Terms: Chemical graph theory; Zagreb index; Randić index; Chemical properties.

1. Introduction

A graph having no loop or multiple edge in known as simple graph. A molecular graph is a simple graph in which atoms and bonds are represented by vertices and edges respectively. The degree of a vertex is the number of edges attached with it. The maximum degree of vertex among the vertices of a graph is denoted by $\Delta(G)$. Kulli [1] introduces the concept of reverse vertex degree c_v, as $c_v = \Delta(G) - d_g(v) + 1$.
In discrete mathematics, graph theory in general, not only the study of different properties of objects but it also tells us about objects having same properties as investigating object [2]. In particular, graph polynomials related to graph are rich in information [3, 4, 5, 6, 7, 8].

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Mathematical tools like polynomials and topological based numbers have significant importance to collect information about properties of chemical compounds [9, 10, 11]. We can find out many hidden information about compounds through theses tools.
Actually, topological indices are numeric quantities that tells us about the whole structure of graph. There are many topological indices [12, 13, 14, 15] that helps us to study physical, chemical reactivities and biological properties. Wiener [16], in 1947, firstly introduce the concept of topological index while working on boiling point. Hosoya polynomial [3] plays an important role in the area of distance-based topological indices. We can find out Wiener index, Hyper Wiener index and Tratch-stankevich-zefirove index from Hosoya polynomial. Randić index defined by Milan Randić [17] in 1975 is one of the oldest degree based topological index and have been extensively studied by mathematician and chemists [18, 19, 20, 21, 22]. Later Gutman *et al.* introduced the first and second Zagreb indices as:

$$M_1(G) = \sum_{uv \in E(G)} (d_u + d_v),$$

$$M_2(G) = \sum_{uv \in E(G)} (d_u . d_v).$$

Zagreb indices help us in finding $\pi-$electronic energy [23]. Many papers [24, 25, 26, 27], surveys [23, 28, 29] and many modification of Zagreb indices are presented in literture [1, 30, 31, 32, 33, 34]. First and second Zagreb polynomials were defined in [7] as:

$$M_1(G, x) = \sum_{uv \in E(G)} x^{(d_u + d_v)},$$

$$M_2(G, x) = \sum_{uv \in E(G)} x^{(d_u . d_v)}.$$

Motivated by these indices, Shirdel *et al.* [35] proposed the first and second hyper Zagreb indices as:

$$H_1(G) = \sum_{uv \in E(G)} (d_u + d_v)^2,$$

$$H_2(G) = \sum_{uv \in E(G)} (d_u . d_v)^2.$$

The first and second Reverse Zagreb indices [1] are defined as:

$$CM_1(G) = \sum_{uv \in E(G)} (c_u + c_v),$$

$$CM_2(G) = \sum_{uv \in E(G)} (c_u . c_v).$$

The first and second Reverse Hyper Zagreb indices [1] are defined as:

$$HCM_1(G) = \sum_{uv \in E(G)} (c_u + c_v)^2,$$

$$HCM_2(G) = \sum_{uv \in E(G)} (c_u.c_v)^2.$$

With the help of reverse Zagreb and hyper Zagreb indices, we now able to define the reverse Zagreb and reverse hyper Zagreb polynomials as:

$$CM_1(G,x) = \sum_{uv \in E(G)} x^{(c_u + c_v)},$$

$$CM_2(G,x) = \sum_{uv \in E(G)} x^{(c_u.c_v)},$$

and

$$HCM_1(G,x) = \sum_{uv \in E(G)} x^{(c_u + c_v)^2},$$

$$HCM_2(G,x) = \sum_{uv \in E(G)} x^{(c_u.c_v)^2}.$$

Till now there are more than 148 topological indices and non of them complete describe all properties of understudy molecular compound, so there is always room to define new topological indices. Our aim is to study Silicon Carbide $Si_2C_3I[r,s]$ and $Si_2C_3II[r,s]$. The multiplicative first and second Zagreb indices, multiplicative hyper Zagreb indices, and some other multiplicative degree-based topological indices of $Si_2C_3I[r,s]$ and $Si_2C_3II[r,s]$ are computed in [36]. Imran *et al.* in [37] computed the general Randić and Zagreb types indices, geometric arithmetic index, atom-bond connectivity index, fourth atom-bond connectivity and fifth geometric arithmetic index of $Si_2C_3I[r,s]$ and $Si_2C_3II[r,s]$. In this report, we aim to compute reversed first and second Zagreb indices, reversed first and second Hyper Zagreb indices, reversed first and second Zagreb polynomials and reversed first and second Hyper Zagreb polynomials for $Si_2C_3I[r,s]$ and $Si_2C_3II(G)$. Figures of this paper are taken from [36, 37].

2. Silicon Carbide $Si_2C_3I[r,s]$ and $Si_2C_3II[r,s]$

In 1891, an American scientist discover Silicon Carbide. But now a days, we can produce silicon carbide artificially by silica and carbon. Till 1929, silicon carbide was known as the hardest material on earth. Its Mohs hardness rating is 9, which makes this similar to diamond. Here, we will find out reverse zagreb, hyper reverse Zagreb and its polynomials for silicon carbide $Si_2C_3I[r,s]$ and $Si_2C_3II[r,s]$.

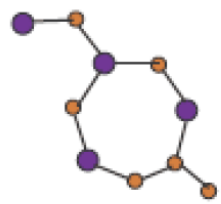

Figure 1. Unit Cell of $Si_2C_3I[r,s]$

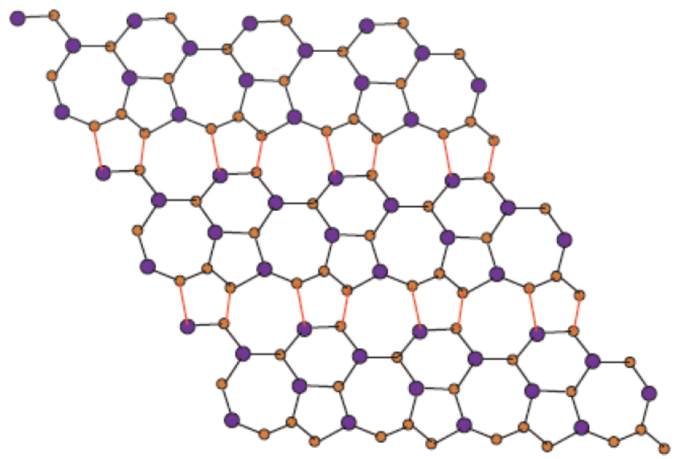

Figure 2. $Si_2C_3I[r,s]$ for $r=4,\ s=3$

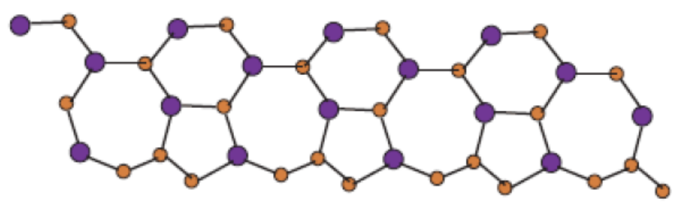

Figure 3. $Si_2C_3I[r,s]$ for $r=4,\ s=1$

3. Main Results

Here, we will compute reverse Zagreb and reverse hyper Zagreb indices for Silicon Carbide $Si_2C_3I[r,s]$ and $Si_2C_3II[r,s]$, where r is the number of connected unit cells, in row (chain) and s is the number of connected rows each with r number of cells.

3.1. Silicon Carbide $Si_2C_3I[r,s]$.

Theorem 3.1. *For the Silicon Carbide $Si_2C_3I[r,s]$, the first and second reverse Zagreb indices are:*

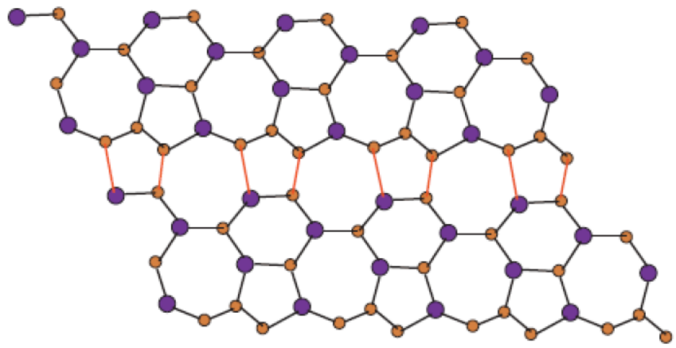

Figure 4. $Si_2C_3I[r,s]$ for $r=4$, $s=2$

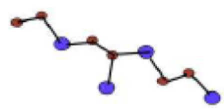

Figure 5. Unit Cell of $Si_2C_3II[r,s]$

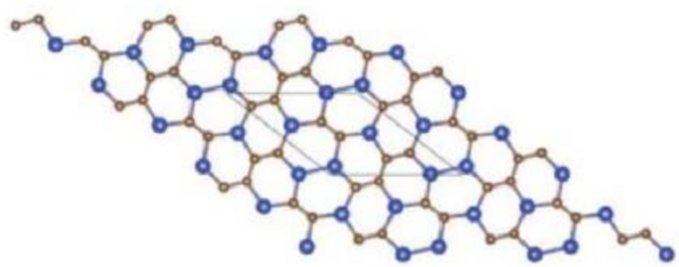

Figure 6. $Si_2C_3II[r,s]$ for $r=3$, $s=3$

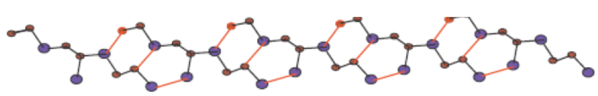

Figure 7. $Si_2C_3II[r,s]$ for $r=5$, $s=1$

(1) $CM_1(Si_2C_3I[r,s]) = 30rs + 4r + 6s - 4$,
(2) $CM_2(Si_2C_3I[r,s]) = 15rs + 7r + 9s - 2$.

Proof. The vertex and edge set of Silicon Carbide is , $|V(Si_2C_3I[r,s])| = 10rs$ and $|E(Si_2C_3I[r,s])| = 15rs - 2r - 3s$, respectively. From Figures 1-4, we can say that, there are five type of edges in $Si_2C_3I[r,s]$. The edge set of $Si_2C_3I[r,s]$ is portioned into the following five edge sets:

$$E_1(Si_2C_3I[r,s]) = \{uv \in E(Si_2C_3I[r,s]); d_u = 1, d_v = 2\},$$

$$E_2(Si_2C_3I[r,s]) = \{uv \in E(Si_2C_3I[r,s]); d_u = 1, d_v = 3\},$$

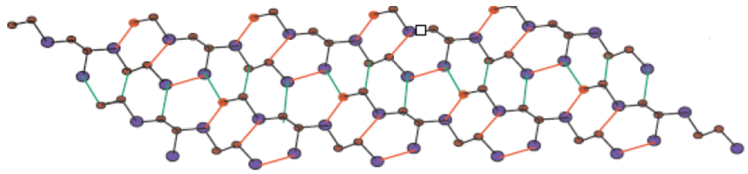

Figure 8. $Si_2C_3II[r,s]$ for $r=5,\ s=2$

$$E_3(Si_2C_3I[r,s]) = \{uv \in E(Si_2C_3I[r,s]); d_u = 2, d_v = 2\},$$
$$E_4(Si_2C_3I[r,s]) = \{uv \in E(Si_2C_3I[r,s]); d_u = 2, d_v = 3\},$$
$$E_5(Si_2C_3I[r,s]) = \{uv \in E(Si_2C_3I[r,s]); d_u = 3, d_v = 3\},$$

such that, $|E_1(Si_2C_3I[r,s])| = 1$,
$|E_2(Si_2C_3I[r,s])| = 1$,
$|E_3(Si_2C_3I[r,s])| = r+2s$,
$|E_4(Si_2C_3I[r,s])| = 6r-1+8(s-1)$
and $|E_5(Si_2C_3I[r,s])| = 15rs-9r-13s+7$.
As, the maximum degree in $Si_2C_3I[r,s]$ is 3, so,

$$c_u = \Delta(G) - d_G(u) + 1 = 4 - d_G(u).$$

The reverse edge set of $Si_2C_3I[r,s]$ is given as:

$$CE_1(Si_2C_3I[r,s]) = \{uv \in E(Si_2C_3I[r,s]); c_u = 3, c_v = 2\},$$
$$CE_2(Si_2C_3I[r,s]) = \{uv \in E(Si_2C_3I[r,s]); c_u = 3, c_v = 1\},$$
$$CE_3(Si_2C_3I[r,s]) = \{uv \in E(Si_2C_3I[r,s]); c_u = 2, c_v = 2\},$$
$$CE_4(Si_2C_3I[r,s]) = \{uv \in E(Si_2C_3I[r,s]); c_u = 2, c_v = 1\},$$
$$CE_5(Si_2C_3I[r,s]) = \{uv \in E(Si_2C_3I[r,s]); c_u = 1, c_v = 1\}.$$

And, $|CE_1(Si_2C_3I[r,s])| = 1$,
$|CE_2(Si_2C_3I[r,s])| = 1$,
$|CE_3(Si_2C_3I[r,s])| = r+2s$,
$|CE_4(Si_2C_3I[r,s])| = 6r-1+8(s-1)$
and $|CE_5(Si_2C_3I[r,s])| = 15rs-9r-13s+7$.

(1) The first reverse Zagreb index for $Si_2C_3I[r,s]$ is:

$$\begin{aligned} CM_1(Si_2C_3I[r,s]) &= \sum_{uv\in E(G)} (c_u + c_v) \\ &= (3+2)(1) + (3+1)(1) + (2+2)(r+2s) + (2+1) \\ &\quad (6r-1+8(s-1)) + (1+1)(15rs-9r-13s+7) \\ &= 30rs + 4r + 6s - 4. \end{aligned}$$

(2) The second reverse Zagreb index for $Si_2C_3I[r,s]$ is:

$$\begin{aligned} CM_2(Si_2C_3I[r,s]) &= \sum_{uv\in E(G)} (c_u . c_v) \\ &= (3\cdot 2)(1) + (3\cdot 1)(1) + (2\cdot 2)(r+2s) + (2\cdot 1)(6r-1 \end{aligned}$$

$$\begin{aligned} & +8(s-1))+(1\cdot 1)(15rs-9r-13s+7) \\ = \;& 15rs+7r+9s-2. \end{aligned}$$

□

Theorem 3.2. *The first and second reverse Zagreb polynomials for $Si_2C_3I[r,s]$ are:*

(1) $CM_1(Si_2C_3I[r,s],x) = x^5+(r+2s+1)x^4+(6r-1+8(s-1))x^3+(15rs-9r-13s+7)x^2$,
(2) $CM_2(Si_2C_3I[r,s],x) = x^6+(r+2s)x^4+x^3+(6r-1+8(s-1))x^2+(15rs-9r-13s+7)x$.

Proof. Now, by the reverse edge partitions for $Si_2C_3I[r,s]$, we have:
(1) The first reverse Zagreb polynomial for $Si_2C_3I[r,s]$, is given as:

$$\begin{aligned} CM_1(Si_2C_3I[r,s],x) &= \sum_{uv\in E(G)} x^{(c_u+c_v)} \\ &= (1)x^{(3+2)}+(1)x^{(3+1)}+(r+2s)x^{(2+2)}+(6r-1 \\ &\quad +8(s-1))x^{(2+1)}+(15rs-9r-13s+7)x^{(1+1)} \\ &= x^5+(r+2s+1)x^4+(6r-1+8(s-1))x^3 \\ &\quad +(15rs-9r-13s+7)x^2. \end{aligned}$$

(2) The second reverse Zagreb polynomial for $Si_2C_3I[r,s]$, is given as:

$$\begin{aligned} CM_2(Si_2C_3I[r,s],x) &= \sum_{uv\in E(G)} x^{(c_u.c_v)} \\ &= (1)x^{(3\cdot 2)}+(1)x^{(3.1)}+(r+2s)x^{(2\cdot 2)}+(6r-1 \\ &\quad +8(s-1))x^{(2\cdot 1)}+(15rs-9r-13s+7)x^{(1\cdot 1)} \\ &= x^6+(r+2s)x^4+x^3+(6r-1+8(s-1))x^2 \\ &\quad +(15rs-9r-13s+7)x. \end{aligned}$$

□

Theorem 3.3. *The first and second reverse hyper Zagreb indices of Silicon Carbon $Si_2C_3I[r,s]$ are:*

(1) $HCM_1(Si_2C_3I[r,s]) = 60rs+34r+50s-12$,
(2) $HCM_2(Si_2C_3I[r,s]) = 15rs+31r+55s-4$.

Proof. By reverse edge partition and definition of reverse hyper Zagreb indices, we have:
(1) The first reverse hyper Zagreb index for $Si_2C_3I[r,s]$ is:

$$\begin{aligned} CM_1(Si_2C_3I[r,s]) &= \sum_{uv\in E(G)} (c_u+c_v)^2 \\ &= (3+2)^2(1)+(3+1)^2(1)+(2+2)^2(r+2s)+(2+1)^2 \\ &\quad (6r-1+8(s-1))+(1+1)^2(15rs-9r-13s+7) \end{aligned}$$

$$= \quad 60rs + 34r + 50s - 12.$$

(2) The second reverse hyper Zagreb index for $Si_2C_3I[r,s]$ is:

$$\begin{aligned} CM_2(Si_2C_3I[r,s]) &= \sum_{uv\in E(G)} (c_u.c_v)^2 \\ &= (3\cdot 2)^2(1) + (3\cdot 1)^2(1) + (2\cdot 2)^2(r+2s) + (2\cdot 1)^2 \\ &\quad (6r-1+8(s-1)) + (1\cdot 1)^2(15rs-9r-13s+7) \\ &= 15rs + 31r + 55s - 4. \end{aligned}$$

□

Theorem 3.4. *The first and second reverse hyper Zagreb polynomials of Si_2C_3 $I[r,s]$ are:*

(1) $HCM_1(Si_2C_3I[r,s],x) = x^{25} + (r+2s+1)x^{16} + (6r-1+8(s-1))x^9 + (15rs-9r-13s+7)x^4$,
(2) $HCM_1(Si_2C_3I[r,s],x) = x^{36} + (r+2s)x^{16} + x^9 + (6r-1+8(s-1))x^4 + (15rs-9r-13s+7)x$.

Proof. Now, by the reverse edge partitions for $Si_2C_3I[r,s]$, we have
(1) The first reverse Zagreb polynomial for $Si_2C_3I[r,s]$, is given as:

$$\begin{aligned} HCM_1(Si_2C_3I[r,s],x) &= \sum_{uv\in E(G)} x^{(c_u+c_v)^2} \\ &= (1)x^{(3+2)^2} + (1)x^{(3+1)^2} + (r+2s)x^{(2+2)^2} + (6r-1 \\ &\quad +8(s-1))x^{(2+1)^2} + (15rs-9r-13s+7)x^{(1+1)^2} \\ &= x^{25} + (r+2s+1)x^{16} + (6r-1+8(s-1))x^9 \\ &\quad +(15rs-9r-13s+7)x^4. \end{aligned}$$

(2) The second reverse Zagreb polynomial for $Si_2C_3I[r,s]$, is given as:

$$\begin{aligned} HCM_2(Si_2C_3I[r,s],x) &= \sum_{uv\in E(G)} x^{(c_u.c_v)^2} \\ &= (1)x^{(3\cdot 2)^2} + (1)x^{(3\cdot 1)^2} + (r+2s)x^{(2\cdot 2)^2} + (6r-1 \\ &\quad +8(s-1))x^{(2\cdot 1)^2} + (15rs-9r-13s+7)x^{(1\cdot 1)^2} \\ &= x^{36} + (r+2s)x^{16} + x^9 + (6r-1+8(s-1))x^4 \\ &\quad +(15rs-9r-13s+7)x. \end{aligned}$$

□

In the following Table 1, we computed first and second reverse Zagreb and first and second reverse hyper Zagreb indices for $Si_2C_3I[r,s]$ for specific values of r and s.

Table 1. first and second reverse Zagreb and first and second reverse hyper Zagreb indices for $Si_2C_3I[r,s]$ for specific values of r and s

	$r=1,s=1$	$r=1,s=2$	$r=2,s=1$	$r=2,s=2$	$r=2,s=3$	$r=3,s=3$
CM_1	36	72	70	136	202	296
CM_2	29	53	51	90	129	181
HCM_1	132	242	226	396	566	780
HCM_2	97	167	143	228	313	389

3.2. Silicon Carbide $Si_2C_3II[r,s]$.

Theorem 3.5. *For the Silicon Carbide $Si_2C_3II[r,s]$, the first and second reverse Zagreb indices are:*

(1) $CM_1(Si_2C_3II[r,s]) = 30rs + 6r + 6s - 6$,
(2) $CM_2(Si_2C_3II[r,s]) = 15rs + 11r + 11s - 2$.

Proof. The vertex and edge set of Silicon Carbide is, $|V(Si_2C_3II[r,s])| = 10rs$ and $|E(Si_2C_3II[r,s])| = 15rs - 2r - 3s$, respectively. From the Figures 5-8, we can observe that, there are five type of edges in $Si_2C_3II[r,s]$. The edge set of $Si_2C_3II[r,s]$ is portioned into following five edge sets:

$$E_1(Si_2C_3II[r,s]) = \{uv \in E(Si_2C_3II[r,s]); d_u = 1, d_v = 2\},$$
$$E_2(Si_2C_3II[r,s]) = \{uv \in E(Si_2C_3II[r,s]); d_u = 1, d_v = 3\},$$
$$E_3(Si_2C_3II[r,s]) = \{uv \in E(Si_2C_3II[r,s]); d_u = 2, d_v = 2\},$$
$$E_4(Si_2C_3II[r,s]) = \{uv \in E(Si_2C_3II[r,s]); d_u = 2, d_v = 3\},$$
$$E_5(Si_2C_3II[r,s]) = \{uv \in E(Si_2C_3II[r,s]); d_u = 3, d_v = 3\},$$

such that, $|E_1(Si_2C_3II[r,s])| = 2$,
$|E_2(Si_2C_3II[r,s])| = 1$,
$|E_3(Si_2C_3II[r,s])| = 2r + 2s$,
$|E_4(Si_2C_3II[r,s])| = 8r + 8s - 14$
and $|E_5(Si_2C_3II[r,s])| = 15rs - 13r - 13s + 11$.
The maximum vertex degree $Si_2C_3II[r,s]$ is 3, so,

$$c_u = \Delta(G) - d_G(u) + 1 = 4 - d_G(u).$$

The reverse edge set of $Si_2C_3II[r,s]$ is given as:

$$CE_1(Si_2C_3II[r,s]) = \{uv \in E(Si_2C_3II[r,s]); c_u = 3, c_v = 2\},$$
$$CE_2(Si_2C_3II[r,s]) = \{uv \in E(Si_2C_3II[r,s]); c_u = 3, c_v = 1\},$$
$$CE_3(Si_2C_3II[r,s]) = \{uv \in E(Si_2C_3II[r,s]); c_u = 2, c_v = 2\},$$
$$CE_4(Si_2C_3II[r,s]) = \{uv \in E(Si_2C_3II[r,s]); c_u = 2, c_v = 1\},$$
$$CE_5(Si_2C_3II[r,s]) = \{uv \in E(Si_2C_3II[r,s]); c_u = 1, c_v = 1\},$$

and we have, $|E_1(Si_2C_3II[r,s])| = 2$,
$|E_2(Si_2C_3II[r,s])| = 1$,
$|E_3(Si_2C_3II[r,s])| = 2r + 2s$,
$|E_4(Si_2C_3II[r,s])| = 8r + 8s - 14$

and $|E_5(Si_2C_3II[r,s])| = 15rs - 13r - 13s + 11$.
(1) The first reverse Zagreb index for $Si_2C_3II[r,s]$ is:

$$\begin{aligned} CM_1(Si_2C_3II[r,s]) &= \sum_{uv \in E(G)} (c_u + c_v) \\ &= (3+2)(2) + (3+1)(1) + (2+2)(2r+2s) + (2+1) \\ &\quad (8r+8s-14) + (1+1)(15rs - 13r - 13s + 11) \\ &= 30rs + 6r + 6s - 6. \end{aligned}$$

(2) The second reverse Zagreb index for $Si_2C_3II[r,s]$ is:

$$\begin{aligned} CM_2(Si_2C_3II[r,s]) &= \sum_{uv \in E(G)} (c_u . c_v) \\ &= (3 \cdot 2)(2) + (3 \cdot 1)(1) + (2 \cdot 2)(2r+2s) + (2 \cdot 1) \\ &\quad (8r+8s-14) + (1 \cdot 1)(15rs - 13r - 13s + 11) \\ &= 15rs + 11r + 11s - 2. \end{aligned}$$

□

Theorem 3.6. *The first and second reverse Zagreb polynomials for $Si_2C_3II[r,s]$ are:*

(1) $CM_1(Si_2C_3II[r,s],x) = x^5 + (2r+2s+1)x^4 + (8r+8s-14)x^3 + (15rs - 9r - 13s + 7)x^2$,
(2) $CM_2(Si_2C_3II[r,s],x) = 2x^6 + (2r+2s)x^4 + x^3 + (8r+8s = 14)x^2 + (15rs - 13r - 13s + 11)x$.

Proof. Now, by the reverse edge partitions for $Si_2C_3II[r,s]$, we have:
(1) The first reverse Zagreb polynomial for $Si_2C_3II[r,s]$, is given as:

$$\begin{aligned} CM_1(Si_2C_3II[r,s],x) &= \sum_{uv \in E(G)} x^{(c_u + c_v)} \\ &= (2)x^{(3+2)} + (1)x^{(3+1)} + (2r+2s)x^{(2+2)} + (8r \\ &\quad +8s-14)x^{(2+1)} + (15rs - 13r - 13s + 11)x^{(1+1)} \\ &= x^5 + (2r+2s+1)x^4 + (8r+8s-14)x^3 \\ &\quad +(15rs - 9r - 13s + 7)x^2. \end{aligned}$$

(2) The second reverse Zagreb polynomial for $Si_2C_3II[r,s]$, is given as:

$$\begin{aligned} CM_2(Si_2C_3II[r,s],x) &= \sum_{uv \in E(G)} x^{(c_u . c_v)} \\ &= (2)x^{(3 \cdot 2)} + (1)x^{(3 \cdot 1)} + (2r+2s)x^{(2 \cdot 2)} + (8r \\ &\quad +8s-14)x^{(2 \cdot 1)} + (15rs - 13r - 13s + 11)x^{(1 . \cdot 1)} \\ &= 2x^6 + (2r+2s)x^4 + x^3 + (8r+8s-14)x^2 \\ &\quad +(15rs - 13r - 13s + 11)x. \end{aligned}$$

□

Theorem 3.7. *The first and second reverse Hyper Zagreb indices of Silicon Carbon $Si_2C_3II[r,s]$ are:*

(1) $HCM_1(Si_2C_3II[r,s]) = 60rs + 51r + 51s - 16,$
(2) $HCM_2(Si_2C_3II[r,s]) = 15rs + 51r + 51s + 108.$

Proof. By reverse edge partition and definition of reverse hyper Zagreb indices, we have:
(1) The first reverse Hyper Zagreb index for $Si_2C_3II[r,s]$ is:

$$\begin{aligned} CM_1(Si_2C_3II[r,s]) &= \sum_{uv\in E(G)} (c_u + c_v)^2 \\ &= (3+2)^2(2) + (3+1)^2(1) + (2+2)^2(2r+2s) + (2+1)^2 \\ &\quad (8r+8s-14) + (1+1)^2(15rs - 13r - 13s + 11) \\ &= 60rs + 52r + 52s - 16. \end{aligned}$$

(2) The second reverse Hyper Zagreb index for $Si_2C_3I[r,s]$ is:

$$\begin{aligned} CM_2(Si_2C_3II[r,s]) &= \sum_{uv\in E(G)} (c_u . c_v)^2 \\ &= (3+2)^2(2) + (3+1)^2(1) + (2+2)^2(2r+2s) \\ &\quad +(2+1)^2(8r+8s-14) + (1+1)^2(15rs \\ &\quad -13r - 13s + 11) \\ &= 15rs + 51r + 51s + 108. \end{aligned}$$

□

Theorem 3.8. *The first and second reverse hyper Zagreb polynomials of Si_2C_3 $II[r,s]$ is:*

(1) $HCM_1(Si_2C_3II[r,s],x) = 2x^{25} + (2r+2s+1)x^{16} + (8r+8s-14)x^9 + (15rs - 13r - 13s + 11)x^4,$
(2) $HCM_1(Si_2C_3II[r,s],x) = 2x^{36} + (2r+2s)x^{16} + x^9 + (8r+8s-14)x^4 + (15rs - 13r - 13s + 11)x.$

Proof. Now, by the reverse edge partitions for $Si_2C_3II[r,s]$, we have:
(1) The first reverse Zagreb polynomial for $Si_2C_3II[r,s]$, is given as:

$$\begin{aligned} CM_1(Si_2C_3II[r,s],x) &= \sum_{uv\in E(G)} x^{(c_u+c_v)^2} \\ &= (2)x^{(3+2)^2} + (1)x^{(3+1)^2} + (2r+2s)x^{(2+2)^2} + (8r \\ &\quad +8s-14)x^{(2+1)^2} + (15rs - 13r - 13s + 11)x^{(1+1)^2} \\ &= 2x^{25} + (2r+2s+1)x^{16} + (8r+8s-14)x^9 \\ &\quad +(15rs - 13r - 13s + 11)x^4. \end{aligned}$$

(2) The second reverse Zagreb polynomial for $Si_2C_3I[r,s]$, is given as:

$$\begin{aligned} CM_2(Si_2C_3II[r,s],x) &= \sum_{uv\in E(G)} x^{(c_u.c_v)^2} \\ &= (2)x^{(3\cdot2)^2} + (1)x^{(3\cdot1)^2} + (2r+2s)x^{(2\cdot2)^2} + (8r+8s \\ &\quad -14)x^{(2\cdot1)^2} + (15rs-13r-13s+11)x^{(1\cdot1)^2} \\ &= 2x^{36} + (2r+2s)x^{16} + x^9 + (8r+8s-14)x^4 \\ &\quad +(15rs-13r-13s+11)x. \end{aligned}$$

□

In the following Table 2, we computed first and second reverse Zagreb and first and second reverse Hyper Zagreb indices for $Si_2C_3II[r,s]$ for spefic values of r and s.

Table 2. first and second reverse Zagreb and first and second reverse Hyper Zagreb indices for $Si_2C_3II[r,s]$ for spefic values of r and s

	$r=1,s=1$	$r=1,s=2$	$r=2,s=1$	$r=2,s=2$	$r=2,s=3$	$r=3,s=3$
CM_1	36	72	72	138	204	300
CM_2	35	61	61	162	143	199
HCM_1	146	257	257	428	599	830
HCM_2	225	291	291	372	453	549

Conclusion

The first and second Zagreb indices are used to compute total π-energy of conjugated molecules. These indices are also useful in the study of anti-inflammatory activities of certain chemical instances, and in elsewhere. In this paper we have obtained reverse Zagreb indices, hyper reverse Zagreb indices and their polynomials for Silicon Carbide $Si_2C_3I[r,s]$ and $Si_2C_3I[r,s]$.

Competing Interests

The authors do not have any competing interests in the manuscript.

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