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Solving Multi-level Multi-objective Fractional Programming Problem with Rough Intervals in the Objective Functions

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Authors' contributions

This work was carried out in collaboration between all authors. Author MSO designed the study, performed the statistical analysis, wrote the protocol. Author FAF wrote the first draft of the manuscript. Authors KRR, OEE and FAF managed the analyses of the study. Authors MSO and OEE managed the literature searches. All authors read and approved the final manuscript.

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Abstract

In this paper multi-level multi-objective fractional programming problem (ML-MOFP) is considered where some or all of its coefficients in the objective function are rough intervals. At the first phase of the solution approach and to avoid the complexity of the problem, two FP problems with interval coefficients will be constructed. One of these problems was a FP problem where all of its coefficients are lower approximations of the rough intervals and the other problem was a FP problem where all of its coefficients are upper approximations of rough intervals. At the second phase, a membership function was constructed to develop a fuzzy goal programming model for obtaining the satisfactory solution of the multi-level multi-objective fractional programming problem. The linearization process introduced by Pal

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et al. [1] will be applied to linearize the membership functions.. Finally, a numerical example will be introduced to illustrate the theoretical results.

Keywords: Multi-level programming; Multi-objective programming; fractional programming; rough intervals programming; Fuzzy goal programming.

MSC2010: *90C05; 90C29; 90C99*

List of Symbols

FP	:	*Fractional programming.*
DM	:	*Decision maker.*
MLMP	:	*Multi-level mathematical programming.*
BLMP	:	*Bi-level mathematical programming.*
ML-MOFP	:	*Multi-level Multi-objective fractional programming.*
LP	:	*Linear programming.*
FLDM	:	*First level decision maker.*
SLDM	:	*Second level decision maker.*
LFPs	:	*Linear fractional programming problems.*
LI	:	*Lower intervals.*
UI	:	*Upper intervals.*
FP1	:	*Fractional programming1.*
FP2	:	*Fractional programming2.*
FP3	:	*Fractional programming3.*
FP4	:	*Fractional programming4.*
FGP		*Fuzzy goal programming.*

1 Introduction

A hierarchical decision structures are common in government policies, competitive economic systems, supply chains, agriculture, bio fuel production, vehicle path planning problems, and so on. These types of problems can be formulated using a multi-level mathematical programming (MLMP) approach. In MLMP problems, one decision maker (DM) is located at each decision level, and objective functions needs to be optimized [2,3,4]. Multi-level optimization is a technique developed to solve decentralized problems with multiple decision-makers in hierarchical organizations [5].

During the past few decades, MLMP [2,3,6] as well as bi-level mathematical programming (BLMP) problems [7,8] have been deeply studied and many methodologies have been established for treating such problems. The uses of the concept of the membership function of fuzzy set theory to multi-level programming problems for satisfactory decision was first presented by Lai [9]. Sakawa et al. [10] developed an interactive fuzzy programming for MLMP with fuzzy parameters. Also, Abo-Sinna and Baky [2] presented balance space approach for multi-level multi-objective programming problems.

In various areas of the real world, the problems are modeled as a multi-objective programming. Many methodologies have been presented for treating such problems [1]. However, the issue of choosing a proper method in a given context is still a subject of active research.

Fractional programming deals with the optimization of one or more ratios of functions subject to set constraints. Recently, fractional programming has become one of the planning tools. It is applied in engineering, business, finance, economics and other disciplines [1,3,8,11]. Computer oriented technique was extended by Helmy et al. [12] to solve a special class of ML-MOFP problems.

Emam [13] presented a bi-level integer non-linear programming problem with linear or non-linear constraints, and in which the non-linear objective function at each level were maximized. It proposed a two planner integer model and a solution method for solving this problem. Therefore Emam proposed an interactive approach for solving bi-level integer multi-objective fractional programming problem [14].

The rough set expressed by a boundary region of a set which is described by lower and upper approximation sets where the set is considered as a crisp set if the boundary region is empty. This is exactly the idea of vagueness [15,16]. The approach for solving rough interval programming problem is to convert the objective function from rough interval to crisp using theorem of crisp evaluation. Roughness is a kind of uncertainty, another kind of uncertainty introduced in [17].

Hamzehee et al. [18] presented a linear programming (LP) problem which is considered where some or all of its coefficients in the objective function and /or constraints are rough intervals. In order to solve this problem, two LP problems with interval coefficients will be constructed. One of these problems is a LP where all of its coefficients are upper approximations of rough intervals and the other problem is a LP where all of its coefficients are lower approximations of rough intervals. Using these two LPs, two newly solutions are defined.

Many researches have been done in the area of rough set and rough intervals [19- 22].

In this paper multi-level multi-objective fractional programming problem is considered when some or all of the coefficients of the objective functions are rough intervals. The remaining of the paper unfolds as follows: Section 2 introduces formulation and solution concept. Section 3, introduces the solution algorithm. In section 4, an illustrative example will be introduced. Finally, in Section 4, conclusion and some open points for future research work are stated in the field of rough intervals multi-level multi-objective fractional programming problems.

2 Problem Formulation and Solution Concept

Multi-level programming problems have more than one decision maker. A decision maker is located at each decision level and a vector of fractional objective functions needs to be optimized. Consider the hierarchical system be composed of a t-level decision makers. Let the decision maker at the i^{th}-level denoted by DM_i controls over the decision variable $\boldsymbol{x_i} = \left(x_{i1}, x_{i2}, \dots, x_{in_i}\right) \in R^{n_i}, i = 1,2, \dots, t$. where $\boldsymbol{x} = (\boldsymbol{x_1}, \boldsymbol{x_2}, \dots, \boldsymbol{x_t}) \in R^n$ and $n = \sum_{i=1}^{t} n_i$.

Mathematically, ML-MOFP problem with rough intervals in the objective functions of maximization-type may be formulated as follows:

$[\boldsymbol{1^{st}\ Level}]$

$$\underbrace{max}_{\boldsymbol{x_1}}\ F_1(\boldsymbol{x}) = \underbrace{max}_{\boldsymbol{x_1}}\left(f_{11}(\boldsymbol{x}), f_{12}(\boldsymbol{x}), \dots, f_{1m_1}(\boldsymbol{x})\right), \tag{1}$$

where $\boldsymbol{x_2}, \boldsymbol{x_3}, \dots, \boldsymbol{x_t}$ *solves*

$[\boldsymbol{2^{nd}\ Level}]$

$$\underbrace{max}_{\boldsymbol{x_2}}\ F_2(\boldsymbol{x}) = \underbrace{max}_{\boldsymbol{x_2}}\left(f_{21}(\boldsymbol{x}), f_{22}(\boldsymbol{x}), \dots, f_{2m_2}(\boldsymbol{x})\right), \tag{2}$$

⋮

where $\boldsymbol{x_t}$ *solves*

[$\boldsymbol{t^{th}}$ ***Level***]

$$\underbrace{max}_{x_t} \; F_t(\boldsymbol{x}) = \underbrace{max}_{x_t} \left(f_{t1}(\boldsymbol{x}), f_{t2}(\boldsymbol{x}), \dots, f_{tm_t}(\boldsymbol{x})\right), \tag{3}$$

subject to

$$x \in G = \left\{ x \in R^n \middle| A_1 x_1 + A_2 x_2 + \dots + A_t x_t \begin{pmatrix} \le \\ = \\ \ge \end{pmatrix} b, x \ge 0, b \in R^m \right\}, \tag{4}$$

where

$$f_{ij}(\boldsymbol{x}) = \frac{N_{ij}(\boldsymbol{x})}{D_{ij}(\boldsymbol{x})} = \frac{\sum_{j=1}^{m_i} \left(\left[\underline{c}_{ij}^L, \underline{c}_{ij}^U\right], \left[\overline{c}_{ij}^L, \overline{c}_{ij}^U\right]\right) x_j + \left(\left[\underline{\alpha}_{ij}^L, \underline{\alpha}_{ij}^U\right], \left[\overline{\alpha}_{ij}^L, \overline{\alpha}_{ij}^U\right]\right)}{\sum_{j=1}^{m_i} d_{ij} x_j + \beta_{ij}}, \quad i = 1,2, \dots, t, \tag{5}$$

$F_1(x)$, $F_2(x)$ and $F_3(x)$ are the objective functions of the first level decision maker (FLDM), second level decision maker (SLDM) and the third level decision maker respectively.

G is the multi-level multi-objective convex constraint set.

$\left(\left[\underline{c}_{ij}^L, \underline{c}_{ij}^U\right], \left[\overline{c}_{ij}^L, \overline{c}_{ij}^U\right]\right)$ are rough intervals coefficients of the objective function,

$\left(\left[\underline{\alpha}_{ij}^L, \underline{\alpha}_{ij}^U\right], \left[\overline{\alpha}_{ij}^L, \overline{\alpha}_{ij}^U\right]\right)$ are rough intervals constants of the numerator.

It is customary to assume that $D_{ij}(\boldsymbol{x}) > 0 \; \forall \; x \in G$, also and β_{ij} are constants of the denominator.

Conversion of (ML-MOFP) problem with rough coefficient in objective functions into upper and lower approximations is usually a hard work for many cases, but transformation process needs to know the following definitions [18]:

Definition 1 [18]:

Rough Interval (RI) can be considered as a qualitative value from vague concept defined on a variable x in R.

Definition 2 [18]:

The qualitative value A is called a rough interval when one can assign two closed intervals A^* and A_* on R to it where $A_* \subseteq A \subseteq A^*$.

Remark 1 [18]:

According to the rough interval properties we have

$\left[\underline{c}_{ij}^L, \underline{c}_{ij}^U\right] \subseteq \left[\overline{c}_{ij}^L, \overline{c}_{ij}^U\right] \rightarrow \quad \overline{c}_{ij}^L \le \underline{c}_{ij}^L \le \underline{c}_{ij}^U \le \overline{c}_{ij}^U,$

$\left[\underline{\alpha}_{ij}^L, \underline{\alpha}_{ij}^U\right] \subseteq \left[\overline{\alpha}_{ij}^L, \overline{\alpha}_{ij}^U\right] \rightarrow \; \overline{\alpha}_{ij}^L \le \underline{\alpha}_{ij}^L \le \underline{\alpha}_{ij}^U \le \overline{\alpha}_{ij}^U,$

Now, the equivalent problems of the (ML-MOFP) problem with rough coefficients in objective functions by using intervals method can be reformulated as follows:

The surely optimal range of ML-MOFP problem (1)-(5) can be gotten by solving the following two classical LFPs:

(The lower intervals in the objective functions (LI))

FP1:

$[\mathbf{1}^{st}\ \boldsymbol{Level}]$

$$\underbrace{max}_{x_1}\ F_1(\boldsymbol{x}) = \underbrace{max}_{x_1}\left(f_{11}(\boldsymbol{x}), f_{12}(\boldsymbol{x}), \dots, f_{1m_1}(\boldsymbol{x})\right), \quad (6)$$

$where\ \boldsymbol{x_2}, \boldsymbol{x_3}, \dots, \boldsymbol{x_t}\ solves$

$[\mathbf{2}^{nd}\ \boldsymbol{Level}]$

$$\underbrace{max}_{x_2}\ F_2(\boldsymbol{x}) = \underbrace{max}_{x_2}\left(f_{21}(\boldsymbol{x}), f_{22}(\boldsymbol{x}), \dots, f_{2m_2}(\boldsymbol{x})\right), \quad (7)$$

⋮

$where\ \boldsymbol{x_t}\ solves$

$[\ \boldsymbol{t^{th}}\ \boldsymbol{Level}]$

$$\underbrace{max}_{x_t}\ F_t(\boldsymbol{x}) = \underbrace{max}_{x_t}\left(f_{t1}(\boldsymbol{x}), f_{t2}(\boldsymbol{x}), \dots, f_{tm_t}(\boldsymbol{x})\right), \quad (8)$$

$subject\ to$

$$x \in G = \left\{x \in R^n \middle| A_1x_1 + A_2x_2 + \dots + A_tx_t \begin{pmatrix} \le \\ = \\ \ge \end{pmatrix} b, x \ge 0, b \in R^m \right\} \quad (9)$$

where

$$f_{ij}(\boldsymbol{x}) = \frac{N_{ij}(\boldsymbol{x})}{D_{ij}(\boldsymbol{x})} = \frac{\sum_{j=1}^{m_i} \underline{c}_{ij}^L x_j + \underline{\alpha}_{ij}^L}{\sum_{j=1}^{m_i} d_{ij}x_j + \beta_{ij}}, i = 1,2, \dots, t. \quad (10)$$

FP2:

$[\mathbf{1}^{st}\ \boldsymbol{Level}]$

$$\underbrace{max}_{x_1}\ F_1(\boldsymbol{x}) = \underbrace{max}_{x_1}\left(f_{11}(\boldsymbol{x}), f_{12}(\boldsymbol{x}), \dots, f_{1m_1}(\boldsymbol{x})\right), \quad (11)$$

$where\ \boldsymbol{x_2}, \boldsymbol{x_3}, \dots, \boldsymbol{x_t}\ solves$

$[\mathbf{2}^{nd}\ \boldsymbol{Level}]$

$$\underbrace{max}_{x_2}\ F_2(\boldsymbol{x}) = \underbrace{max}_{x_2}\left(f_{21}(\boldsymbol{x}), f_{22}(\boldsymbol{x}), \dots, f_{2m_2}(\boldsymbol{x})\right), \quad (12)$$

⋮

$where\ \boldsymbol{x_t}\ solves$

$[\ \boldsymbol{t^{th}}\ \boldsymbol{Level}]$

$$\underbrace{max}_{x_t}\ F_t(\boldsymbol{x}) = \underbrace{max}_{x_t}\left(f_{t1}(\boldsymbol{x}), f_{t2}(\boldsymbol{x}), \dots, f_{tm_t}(\boldsymbol{x})\right), \quad (13)$$

$subject\ to$

$$x \in G = \left\{x \in R^n \middle| A_1x_1 + A_2x_2 + \dots + A_tx_t \begin{pmatrix} \le \\ = \\ \ge \end{pmatrix} b, x \ge 0, b \in R^m \right\} \quad (14)$$

Where

$$f_{ij}(\boldsymbol{x}) = \frac{N_{ij}(\boldsymbol{x})}{D_{ij}(\boldsymbol{x})} = \frac{\sum_{j=1}^{m_i} \underline{c}_{ij}^U x_j + \underline{\alpha}_{ij}^U}{\sum_{j=1}^{m_i} d_{ij}x_j + \beta_{ij}}, i = 1,2, \dots, t. \quad (15)$$

While the possibly optimal range of ML-MOFP problem (1)-(5) can be gotten by solving the following two classical LFPs:

(The upper intervals in the objective functions (UI)

FP3:

$[\mathbf{1}^{st}\ \boldsymbol{Level}]$

$$\underbrace{max}_{x_1}\ F_1(\boldsymbol{x}) =$$

FP4:

$[\mathbf{1}^{st}\ \boldsymbol{Level}]$

$$\underbrace{max}_{x_1}\ F_1(\boldsymbol{x}) =$$

FP3:

$$\underbrace{max}_{x_1}\left(f_{11}(\boldsymbol{x}), f_{12}(\boldsymbol{x}), \dots, f_{1m_1}(\boldsymbol{x})\right), \quad (16)$$

$where\ \boldsymbol{x_2}, \boldsymbol{x_3}, \dots, \boldsymbol{x_t}\ solves$

$[\boldsymbol{2^{nd}\ Level}]$

$$\underbrace{max}_{x_2}\ F_2(\boldsymbol{x}) = \underbrace{max}_{x_2}\left(f_{21}(\boldsymbol{x}), f_{22}(\boldsymbol{x}), \dots, f_{2m_2}(\boldsymbol{x})\right), \quad (17)$$

$\vdots$

$where\ \boldsymbol{x_t}\ solves$

$[\ \boldsymbol{t^{th}\ Level}]$

$$\underbrace{max}_{x_t}\ F_t(\boldsymbol{x}) = \underbrace{max}_{x_t}\left(f_{t1}(\boldsymbol{x}), f_{t2}(\boldsymbol{x}), \dots, f_{tm_t}(\boldsymbol{x})\right), \quad (18)$$

$subject\ to$

$$x \in G = \left\{x \in R^n \middle| \begin{matrix} A_1x_1 + A_2x_2 + \cdots + A_tx_t \begin{pmatrix} \leq \\ = \\ \geq \end{pmatrix} b, \boldsymbol{x} \geq 0, \\ b \in R^m \end{matrix} \right\} \quad (19)$$

where

$$f_{ij}(x) = \frac{N_{ij}(x)}{D_{ij}(x)} = \frac{\sum_{j=1}^{m_i} \overline{c}_{ij}^L x_j + \overline{\alpha}_{ij}^L}{\sum_{j=1}^{m_i} d_{ij}x_j + \beta_{ij}}, i = 1,2, \dots, t. \quad (20)$$

FP4:

$$\underbrace{max}_{x_1}\left(f_{11}(\boldsymbol{x}), f_{12}(\boldsymbol{x}), \dots, f_{1m_1}(\boldsymbol{x})\right), \quad (21)$$

$where\ \boldsymbol{x_2}, \boldsymbol{x_3}, \dots, \boldsymbol{x_t}\ solves$

$[\boldsymbol{2^{nd}\ Level}]$

$$\underbrace{max}_{x_2}\ F_2(\boldsymbol{x}) = \underbrace{max}_{x_2}\left(f_{21}(\boldsymbol{x}), f_{22}(\boldsymbol{x}), \dots, f_{2m_2}(\boldsymbol{x})\right), \quad (22)$$

$\vdots$

$where\ \boldsymbol{x_t}\ solves$

$[\ \boldsymbol{t^{th}\ Level}]$

$$\underbrace{max}_{x_t}\ F_t(\boldsymbol{x}) = \underbrace{max}_{x_t}\left(f_{t1}(\boldsymbol{x}), f_{t2}(\boldsymbol{x}), \dots, f_{tm_t}(\boldsymbol{x})\right), \quad (23)$$

$subject\ to$

$$x \in G = \left\{x \in R^n \middle| \begin{matrix} A_1x_1 + A_2x_2 + \cdots + A_tx_t \begin{pmatrix} \leq \\ = \\ \geq \end{pmatrix} b, \boldsymbol{x} \geq 0, \\ b \in R^m \end{matrix} \right\} \quad (24)$$

Where

$$f_{ij}(\boldsymbol{x}) = \frac{N_{ij}(\boldsymbol{x})}{D_{ij}(\boldsymbol{x})} = \frac{\sum_{j=1}^{m_i} \overline{c}_{ij}^U x_j + \overline{\alpha}_{ij}^U}{\sum_{j=1}^{m_i} d_{ij}x_j + \beta_{ij}}, i = 1,2, \dots, t. \quad (25)$$

For solving the previous classical four (ML-MOFP) problems simultaneously, the fuzzy goal programming approach will be applied. The linearization procedure introduced by pal et al. [1] will be applied to linearize the membership goals.

2.1 Fuzzy goal programming approach for (ML-MOFP) problems

The vector of objective functions for each decision maker is formulated as a fuzzy goal characterized by the membership functions $\mu_{(f_{ij})}$, $(i = 1,2, \dots, t)$, $(j = 1,2, \dots, m_i)$, at each level.

2.1.1 Characterization of membership functions

To define the membership functions of the fuzzy goals each objective function's individual maximum is taken as the corresponding aspiration level, as follows [3,4]:

$$u_{ij} = \underbrace{max}_{x \in G}\left(f_{ij}(\boldsymbol{x})\right), \quad (i = 1,2, \dots, t), (\ j = 1,2, \dots, m_i). \quad (26)$$

where $u_{ij}, (i = 1,2, \dots, t)$, $(j = 1,2, \dots, m_i)$, give the upper tolerance limit or aspiration level of achievement for the membership function of ij^{th} objective function. Similarly, each objective function's individual minimum is taken as the corresponding aspiration level, as follows:

$$g_{ij} = \underbrace{min}_{x \in G} \left(f_{ij}(\boldsymbol{x})\right), \quad (i = 1,2, \dots, t), (j = 1,2, \dots, m_i). \tag{27}$$

where g_{ij}, $(i = 1,2, \dots, t)$, $(j = 1,2, \dots, m_i)$, give the lower tolerance limit or lowest acceptable level of achievement for the membership function of ij^{th} objective function. It can be assumed reasonably the values of $\left(f_{ij}(\boldsymbol{x})\right) \geq u_{ij}$, $(i = 1,2, \dots, t)$, $(j = 1,2, \dots, m_i)$, are acceptable and all values less than $g_{ij} = \underbrace{min}_{x \in G} \left(f_{ij}(\boldsymbol{x})\right)$, are absolutely unacceptable. Then, the membership function $\mu_{ij}\left(f_{ij}(\boldsymbol{x})\right)$, as shown in Fig(1.a), for the ij^{th} fuzzy goal can be formulated as [4]:

$$\mu_{f_{ij}}\left(f_{ij}(\boldsymbol{x})\right) = \begin{cases} 1, & if \ \left(f_{ij}(\boldsymbol{x})\right) \geq u_{ij}, \\ \dfrac{\left(f_{ij}(\boldsymbol{x})\right) - g_{ij}}{u_{ij} - g_{ij}}, & if \ g_{ij} \leq \left(f_{ij}(\boldsymbol{x})\right) \leq u_{ij}, \\ 0, & if \ \left(f_{ij}(\boldsymbol{x})\right) \leq g_{ij}, \end{cases} \quad (i = 1,2, \dots, t), (j = 1,2, \dots, m_i), \tag{28}$$

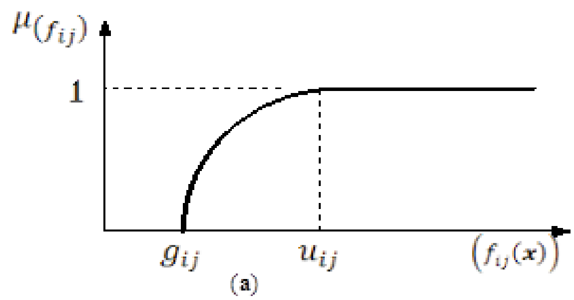

Fig. 1. $(\boldsymbol{a})$ **membership functions of** $\left(\boldsymbol{f_{ij}}(\boldsymbol{x})\right)$

2.2 Fuzzy goal programming methodology

In the decision-making context, each decision maker is interested in maximizing his or her own objective function; the optimal solution of each DM, when calculated in isolation, would be considered as the best solution and the associated value of the objective function can be considered as the aspiration level of the corresponding fuzzy goal. In fuzzy programming approach, the highest degree of membership is one. For the defined membership functions in equation (28), the flexible membership goals having the aspired level unity can be represented as follows:

$$\mu_{f_{ij}}\left(f_{ij}(\boldsymbol{x})\right) + d_{ij}^{-} - d_{ij}^{+} = 1, \quad (i = 1,2, \dots, t), \ (j = 1,2, \dots, m_i), \tag{29}$$

or equivalently as:

$$\frac{\left(f_{ij}(\boldsymbol{x})\right) - g_{ij}}{u_{ij} - g_{ij}} + d_{ij}^{-} - d_{ij}^{+} = 1, \quad (i = 1,2, \dots, t), \ (j = 1,2, \dots, m_i), \tag{30}$$

where $d_{ij}^{-}, d_{ij}^{+} \geq 0$ with $d_{ij}^{-}\, d_{ij}^{+} = 0$, $(i = 1,2, \dots, t)$, $(j = 1,2, \dots, m_i)$ represent the under- and over-deviations, respectively, from the aspired levels [3]:

In the methodology of goal programming, the under- and over- deviational variables are included in the achievement function for minimizing them depends on the type of the objective functions to be optimized. In the proposed FGP approach, the sum of under deviational variables is required to be minimized to achieve the aspired level. It may be noted that any over-deviation from a fuzzy goal indicates the full achievement of the membership value [3]. The equivalent proposed final (ML-MOFP) model of the problem can be formulated as follows:

$$min \;\; Z = \sum_{j=1}^{m_1} w_{1j}^{-}\, d_{1j}^{-} + \sum_{j=1}^{m_2} w_{2j}^{-} d_{2j}^{-} + \cdots + \sum_{j=1}^{m_t} w_{tj}^{-} d_{tj}^{-}\,, \tag{31}$$

subject to

$$\frac{\left(f_{ij}(\boldsymbol{x})\right) - g_{ij}}{u_{ij} - g_{ij}} + d_{ij}^{-} - d_{ij}^{+} = 1, \qquad (i = 1,2, \dots, t), (\, j = 1,2, \dots, m_i), \tag{32}$$

$$x_{ik} = x_{ik}^{*}, \qquad (i = 1,2, \dots, t-1), \;\; (k = 1,2, \dots, n_i), \tag{33}$$

$$x \in G = \left\{ x \in R^n \middle| A_1 x_1 + A_2 x_2 + \cdots + A_t x_t \begin{pmatrix} \leq \\ = \\ \geq \end{pmatrix} b, \boldsymbol{x} \geq 0\,,\;\; b \in R^m \right\} \tag{34}$$

$$d_{ij}^{-}\, d_{ij}^{+} = 0, and\; d_{ij}^{-}, d_{ij}^{+} \geq 0, \;\; (i = 1,2, \dots, t), \;\; (j = 1,2, \dots, m_i), \tag{35}$$

where Z represents the achievement function consisting of the weighted under-deviational variables of the fuzzy goals. The numerical weights w_{ij}^{-} represent the relative importance of achieving the aspired levels of the respective fuzzy goals. To assess the relative importance of the fuzzy goals properly, the values of w_{ij}^{-} are determined as [3]:

$$w_{ij}^{-} = \frac{1}{u_{ij} - g_{ij}}, \qquad (i = 1,2, \dots, t), (\, j = 1,2, \dots, m_i), \tag{36}$$

2.3 Linearization of membership goals

It can be easily noted that the membership goals in equations (32) are nonlinear in nature and this may needs difficult computational in the solution process. To avoid these problems, a linearization procedure is presented in this section [1]. The linearization process for the membership goals in (32) considering the expression of $f_{ij}(\boldsymbol{x})$ in equation (5) will be firstly introduced.

The ij^{th} membership goals can be presented as:

$$\mu_{f_{ij}}\left(f_{ij}(\boldsymbol{x})\right) + d_{ij}^{-} - d_{ij}^{+} = 1, \tag{37}$$

$$L_{ij}\left(f_{ij}(\boldsymbol{x})\right) - L_{ij} g_{ij} + d_{ij}^{-} - d_{ij}^{+} = 1, \quad where \quad L_{ij} = \frac{1}{u_{ij} - g_{ij}}, \tag{38}$$

$$f_{ij}(\boldsymbol{x}) = \frac{N_{ij}(\boldsymbol{x})}{D_{ij}(\boldsymbol{x})} = \frac{\sum_{j=1}^{n} \underline{c}_{ij}^{L} x_j + \underline{\alpha}_{ij}^{L}}{\sum_{j=1}^{n} d_{ij} x_j + \beta_{ij}}, i = 1,2, \dots, t.$$

using the expression of $f_{ij}(x)$, the above goal in equation (38) can be presented as:

$$L_{ij}\frac{(\underline{\boldsymbol{c}}_{ij}^{L})\boldsymbol{x}+\underline{\alpha}_{ij}^{L}}{(\boldsymbol{d}_{ij})\boldsymbol{x}+\beta_{ij}}-L_{ij}g_{ij}+d_{ij}^{-}-d_{ij}^{+}=1, \tag{39}$$

$$L_{ij}\left[(\underline{\boldsymbol{c}}_{ij}^{L})\boldsymbol{x}+\underline{\alpha}_{ij}^{L}\right]-L_{ij}g_{ij}\left[(\boldsymbol{d}_{ij})\boldsymbol{x}+\beta_{ij}\right]+d_{ij}^{-}\left[(\boldsymbol{d}_{ij})\boldsymbol{x}+\beta_{ij}\right]-d_{ij}^{+}\left[(\boldsymbol{d}_{ij})\boldsymbol{x}+\beta_{ij}\right]=\left[(\boldsymbol{d}_{ij})\boldsymbol{x}+\beta_{ij}\right],$$

$$L_{ij}\left[(\underline{\boldsymbol{c}}_{ij}^{L})\boldsymbol{x}+\underline{\alpha}_{ij}^{L}\right]+d_{ij}^{-}\left[(\boldsymbol{d}_{ij})\boldsymbol{x}+\beta_{ij}\right]-d_{ij}^{+}\left[(\boldsymbol{d}_{ij})\boldsymbol{x}+\beta_{ij}\right]=\left(1+L_{ij}g_{ij}\right)\left[(\boldsymbol{d}_{ij})\boldsymbol{x}+\beta_{ij}\right],$$

$$L_{ij}\left[(\underline{\boldsymbol{c}}_{ij}^{L})\boldsymbol{x}+\underline{\alpha}_{ij}^{L}\right]+d_{ij}^{-}\left[(\boldsymbol{d}_{ij})\boldsymbol{x}+\beta_{ij}\right]-d_{ij}^{+}\left[(\boldsymbol{d}_{ij})\boldsymbol{x}+\beta_{ij}\right]=L_{ij}^{0}\left[(\boldsymbol{d}_{ij})\boldsymbol{x}+\beta_{ij}\right],$$

where $L_{ij}^{0}=\left(1+L_{ij}g_{ij}\right)$,

$$\left[L_{ij}(\underline{\boldsymbol{c}}_{ij}^{L})-L_{ij}^{0}(\boldsymbol{d}_{ij})\right]\boldsymbol{x}+d_{ij}^{-}\left[(\boldsymbol{d}_{ij})\boldsymbol{x}+\beta_{ij}\right]-d_{ij}^{+}\left[(\boldsymbol{d}_{ij})\boldsymbol{x}+\beta_{ij}\right]=\left[L_{ij}^{0}\left(\beta_{ij}\right)-L_{ij}\left(\underline{\alpha}_{ij}^{L}\right)\right],$$

$$\underline{\boldsymbol{C}}_{ij}^{L}\boldsymbol{x}+d_{ij}^{-}\left[(\boldsymbol{d}_{ij})\boldsymbol{x}+\beta_{ij}\right]-d_{ij}^{+}\left[(d_{ij})\boldsymbol{x}+\beta_{ij}\right]=\underline{\boldsymbol{G}}_{ij}^{L}, \tag{40}$$

Where

$$\underline{\boldsymbol{C}}_{ij}^{L}=\left[L_{ij}(\underline{\boldsymbol{c}}_{ij}^{L})-L_{ij}^{0}(\boldsymbol{d}_{ij})\right] and \tag{41a}$$

$$\underline{\boldsymbol{G}}_{ij}^{L}=\left[L_{ij}^{0}\left(\beta_{ij}\right)-L_{ij}\left(\underline{\alpha}_{ij}^{L}\right)\right],(i=1,2,\ldots,t),(j=1,2,\ldots,m_{i}) \tag{41b}$$

Thus, considering the method of variable change presented by Pal et al. [1] the goal expression in equation (39) can be linearized as follows.

By setting,

$$D_{ij}^{-}=d_{ij}^{-}\left[(d_{ij})\boldsymbol{x}+\beta_{ij}\right] \; and \; D_{ij}^{+}=d_{ij}^{+}\left[(d_{ij})\boldsymbol{x}+\beta_{ij}\right], \tag{42}$$

Then the linear form of expression in equation (40) is obtained as:

$$(\underline{\boldsymbol{C}}_{ij}^{L})\boldsymbol{x}+D_{ij}^{-}-D_{ij}^{+}=\underline{\boldsymbol{G}}_{ij}^{L}, \tag{43}$$

with $D_{ij}^{-},D_{ij}^{+}\geq 0;\; and\; D_{ij}^{-}\,D_{ij}^{+}=0$ since $d_{ij}^{-},d_{ij}^{+}\geq 0$ and $(\boldsymbol{d}_{ij})\boldsymbol{x}+\beta_{ij}>0$. Now, it is noted that, minimization of d_{ij}^{-} means minimization of $D_{ij}^{-}=d_{ij}^{-}\left[(\boldsymbol{d}_{ij})\boldsymbol{x}+\beta_{ij}\right]$ which is also nonlinear. It may be noted that when the membership goal is fully achieved, $d_{ij}^{-}=0$, and when its achievement is zero, $d_{ij}^{-}=1$, are found in the solution [2,19]. So, involvement of $d_{ij}^{-}\leq 1$, in the solution leads to impose the following constraint in the model of the problem:

$$\frac{D_{ij}^{-}}{\left[(\boldsymbol{d}_{ij})\boldsymbol{x}+\beta_{ij}\right]}\leq 1. \tag{44}$$

Now, based on the simplest version of goal programming, the final proposed FGP model of the **(FP1)** becomes:

$$min\ \ Z = \sum_{j=1}^{m_1} w_{1j}^- D_{1j}^- + \sum_{j=1}^{m_2} w_{2j}^- D_{2j}^- + \cdots + \sum_{j=1}^{m_t} w_{tj}^- D_{tj}^- , \tag{45}$$

$subject\ to$

$$\underline{\boldsymbol{C}}_{ij}^L \boldsymbol{x} + D_{ij}^- - D_{ij}^+ = \underline{\boldsymbol{C}}_{ij}^L, \qquad (i = 1,2, \dots, t), (j = 1,2, \dots, m_i), \tag{46}$$

$$\boldsymbol{x_{ik}} = \boldsymbol{x_{ik}^*}, \qquad (i = 1,2, \dots, t-1), \ \ (k = 1,2, \dots, n_i), \tag{47}$$

$$-(d_{ij})\boldsymbol{x} + D_{ij}^- \leq \beta_{ij}, \qquad (i = 1,2, \dots, t), \ \ (j = 1,2, \dots, m_i), \tag{48}$$

$$x \in G = \left\{ x \in R^n \middle| A_1 x_1 + A_2 x_2 + \cdots + A_t x_t \begin{pmatrix} \leq \\ = \\ \geq \end{pmatrix} b, \boldsymbol{x} \geq 0 \ , \ b \in R^m \right\} \tag{49}$$

$$D_{ij}^-, D_{ij}^+ \geq 0, \qquad (i = 1,2, \dots, t), \ (j = 1,2, \dots, m_i), \tag{50}$$

Similarly, applying the linearization process of the membership goals considering the expression of $f_{ij}(\boldsymbol{x})$ in equations (15),(20) and (25).

3 Solution Algorithm

Step (1): reformulate problem (1)-(5) into (FP1), (FP2), (FP3) and (FP4).

Step (2): For problem (FP1), Compute $u_{ij}, g_{ij}, w_{ij}^-, i = 1,2, \dots, t, j = 1, \dots, m_i$.

Step (3): Construct the membership function$\mu_{ij}\left(f_{ij}(\boldsymbol{x})\right), i = 1,2, \dots, t, j = 1, \dots, m_i$.

Step (4): Compute $\underline{\boldsymbol{C}}_{ij}^L and\ \ \underline{\boldsymbol{G}}_{ij}^L, i = 1,2, \dots, t, j = 1, \dots, m_i$ according to equation (41a), (41b).

Step (5): Do the linearization process for $\mu_{ij}\left(f_{ij}(\boldsymbol{x})\right)$ according to equation (43).

Step (6): Put $i = 1$ in **FGP** model (45)-(50).

Step (7): Solve **FGP** model (45)-(50) to get $x_{1k} = x_{1k}^*, k = 1,2, \dots, n_i$.

Step (8): put $i = i + 1$ in **FGP** model (45)-(50) and go to step (7).

Step (9): If $i > t - 1$, go to step (10), otherwise go to step (8).

Step (10): Solve **FGP** model (45)-(50) with $x_{ik} = x_{ik}^*, i = 1,2, \dots, t-1, k = 1,2, \dots, n_i$.

Step (11): If the DM solves (FP2), (FP3), and (FP4) go to step 13, otherwise go to step12.

Step (12): Repeat steps from (2) to (10) for (FP2), (FP3), and (FP4).

Step (13): Define the surely and possibly optimal range for problem (1)-(5).

Step (14): Stop.

4 An Illustrative Example

To demonstrate the proposed FGP approach, consider the following $(ML - MOFP)$ problem with rough intervals in the objective functions.

$[\mathbf{1}^{st}\ \boldsymbol{Level}\ ]$

$$\underbrace{max}_{x_1}\begin{pmatrix} f_{11} = \dfrac{2([2,3],[1,5])x_1 + ([3,5],[2,7])x_2 + x_3 + ([2,3],[1,4])}{2x_1 + x_2 + x_3 + 1}, \\ f_{12} = \dfrac{([6,7],[5,9])x_1 - x_2 + ([1,3],[1,6])x_3 + ([1,3],[0,5])}{x_2 + x_3 + 3} \end{pmatrix},$$

$where\ \boldsymbol{x_2}, \boldsymbol{x_3} solves$

$[\mathbf{2}^{nd}\ \boldsymbol{Level}]$

$$\underbrace{max}_{x_2}\begin{pmatrix} f_{21} = \dfrac{2x_1 + ([5,6],[3,8])x_2 - 2([0,3],[0,6])x_3 + ([5,6],[3,7])}{x_1 + x_3 + 4}, \\ f_{22} = \dfrac{x_1 - ([3,4],[2,6])x_2 + ([1,3],[1,7])x_3 + ([3,4],[2,6])}{2x_1 + x_3 + 6} \end{pmatrix},$$

$where\ \boldsymbol{x_3} solves$

$[\mathbf{3}^{rd}\ \boldsymbol{Level}]$

$$\underbrace{max}_{x_3}\begin{pmatrix} f_{31} = \dfrac{([2,5],[1,8])x_1 - 2x_2 + x_3 + ([4,5],[3,6])}{x_3 + 2}, \\ f_{32} = \dfrac{5x_1 + 2([1,2],[1,4])x_2 - x_3 + ([6,7],[5,8])}{x_1 + 3x_2 + x_3 + 7} \end{pmatrix},$$

$subject\ to$

$$3x_1 + 5x_2 + x_3 \leq 35,$$

$$2x_1 - x_2 + 12x_3 \leq 20,$$

$$5x_2 + 6x_3 \leq 16\ ,$$

$$x_1, x_2, x_3 \geq 0.$$

For solving the previous example, it will be reformulated into lower intervals problems and upper intervals problems as follows [18]:

(The lower intervals coefficients (LI))

FP1:

$[\mathbf{1^{st}}\ \boldsymbol{Level}]$

$$\underbrace{max}_{x_1}\left(\frac{4x_1+3x_2+x_3+2}{2x_1+x_2+x_3+1},\frac{6x_1-x_2+x_3+1}{x_2+x_3+3}\right),$$

$where\ \boldsymbol{x_2},\boldsymbol{x_3}\ \ solves$

$[\mathbf{2^{nd}}\ \boldsymbol{Level}]$

$$\underbrace{max}_{x_2}\left(\frac{2x_1+5x_2+5}{x_1+x_3+4},\frac{x_1-3x_2+x_3+3}{2x_1+x_3+6}\right)$$

$where\ \boldsymbol{x_3}\ \ solves$

$$\underbrace{max}_{x_3}\left(\frac{2x_1-2x_2+x_3+4}{x_3+2},\frac{5x_1+2x_2-x_3+6}{x_1+3x_2+x_3+7}\right)$$

$subject\ to$

$3x_1+5x_2+x_3\le 35,$
$2x_1-x_2+12x_3\le 20,$

$5x_2+6x_3\le 16\ ,$

$x_1,x_2,x_3\ge 0.$

FP2:

$[\mathbf{1^{st}}\ \boldsymbol{Level}]$

$$\underbrace{max}_{x_1}\left(\frac{6x_1+5x_2+x_3+3}{2x_1+x_2+x_3+1},\frac{7x_1-x_2+3x_3+3}{x_2+x_3+3}\right),$$

$where\ \boldsymbol{x_2},\boldsymbol{x_3}\ \ solves$

$[\mathbf{2^{nd}}\ \boldsymbol{Level}]$

$$\underbrace{max}_{x_2}\left(\frac{2x_1+6x_2-6x_3+6}{x_1+x_3+4},\frac{x_1-4x_2+3x_3+4}{2x_1+x_3+6}\right)$$

$where\ \boldsymbol{x_3}\ \ solves$

$$\underbrace{max}_{x_3}\left(\frac{5x_1-2x_2+x_3+5}{x_3+2},\frac{5x_1+4x_2-x_3+7}{x_1+3x_2+x_3+7}\right)$$

$subject\ to$

$3x_1+5x_2+x_3\le 35,$
$2x_1-x_2+12x_3\le 20,$

$5x_2+6x_3\le 16\ ,$

$x_1,x_2,x_3\ge 0.$

(The upper intervals coefficients (UI)

FP3:

$[\mathbf{1^{st}}\ \boldsymbol{Level}]$

$$\underbrace{max}_{x_1}\left(\frac{2x_1+2x_2+x_3+1}{2x_1+x_2+x_3+1},\frac{5x_1-x_2+x_3}{x_2+x_3+3}\right),$$

$where\ \boldsymbol{x_2},\boldsymbol{x_3}\ \ solves$

$[\mathbf{2^{nd}}\ \boldsymbol{Level}]$

$$\underbrace{max}_{x_2}\left(\frac{2x_1+3x_2+3}{x_1+x_3+4},\frac{x_1-2x_2+x_3+2}{2x_1+x_3+6}\right)$$

$where\ \boldsymbol{x_3}\ \ solves$

$$\underbrace{max}_{x_3}\left(\frac{x_1-2x_2+x_3+3}{x_3+2},\frac{5x_1+2x_2-x_3+5}{x_1+3x_2+x_3+7}\right)$$

$subject\ to$

FP4:

$[\mathbf{1^{st}}\ \boldsymbol{Level}]$

$$\underbrace{max}_{x_1}\left(\frac{10x_1+7x_2+x_3+4}{2x_1+x_2+x_3+1},\frac{9x_1-x_2+6x_3+5}{x_2+x_3+3}\right),$$

$where\ \boldsymbol{x_2},\boldsymbol{x_3}\ \ solves$

$[\mathbf{2^{nd}}\ \boldsymbol{Level}]$

$$\underbrace{max}_{x_2}\left(\frac{2x_1+8x_2-12x_3+7}{x_1+x_3+4},\frac{x_1-6x_2+7x_3+6}{2x_1+x_3+6}\right)$$

$where\ \boldsymbol{x_3}\ \ solves$

$$\underbrace{max}_{x_3}\left(\frac{8x_1-2x_2+x_3+6}{x_3+2},\frac{5x_1+8x_2-x_3+8}{x_1+3x_2+x_3+7}\right)$$

$subject\ to$

$3x_1 + 5x_2 + x_3 \leq 35,$	$3x_1 + 5x_2 + x_3 \leq 35,$
$2x_1 - x_2 + 12x_3 \leq 20,$	$2x_1 - x_2 + 12x_3 \leq 20,$
$5x_2 + 6x_3 \leq 16$,	$5x_2 + 6x_3 \leq 16$,
$x_1, x_2, x_3 \geq 0.$	$x_1, x_2, x_3 \geq 0.$

For solving (FP1), the individual maximum and minimum values are summarized in Table 1. The decided aspiration levels, upper tolerance limits and the weights w_{ij} are also considered.

Table 1. Individual maximum, minimum values, u_{ij}, g_{ij} and weights w_{ij}.

	$f_{11}(\boldsymbol{x})$	$f_{12}(\boldsymbol{x})$	$f_{21}(\boldsymbol{x})$	$f_{22}(\boldsymbol{x})$	$f_{31}(\boldsymbol{x})$	$f_{32}(\boldsymbol{x})$
$max\left(f_{ij}(\boldsymbol{x})\right)$	2.761905	20.3333	5.25	0.6086	12	3.29411
$min\left(f_{ij}(\boldsymbol{x})\right)$	1.375	-0.354838	0.882353	-1.1	0.4	0.5
u_{ij}	2.7	20	5	0.6	12	3.2
g_{ij}	1.3	-0.35	0.88	-1	0.4	0.5
w_{ij}	0.714	0.094	0.243	0.625	0.086	0.37

The coefficient of the linearized membership goals are presented in Table 2.

Table 2. The coefficient of the linearized membership goals $(\boldsymbol{C}^{ij})^T$ *and* $\boldsymbol{G}_{ij}$

	$f_{11}(\boldsymbol{x})$	$f_{12}(\boldsymbol{x})$	$f_{21}(\boldsymbol{x})$	$f_{22}(\boldsymbol{x})$	$f_{31}(\boldsymbol{x})$	$f_{32}(\boldsymbol{x})$
$\left(\underline{\boldsymbol{C}}_{ij}^L\right)^T$	$\begin{pmatrix}-1\\0.214\\-1.214\end{pmatrix}^T$	$\begin{pmatrix}0.294\\-1.029\\-0.931\end{pmatrix}^T$	$\begin{pmatrix}-0.728\\1.215\\0.001\end{pmatrix}^T$	$\begin{pmatrix}-1.25\\-1.875\\-2.25\end{pmatrix}^T$	$\begin{pmatrix}0.172\\-0.172\\-0.948\end{pmatrix}^T$	$\begin{pmatrix}0.665\\-2.815\\-1.555\end{pmatrix}^T$
$\underline{\boldsymbol{G}}_{ij}^L$	0.5	2.891	3.641	0.375	1.724	6.075

4.1 Solving the 1st level FGP model

$min\ Z = 0.714D_{11}^- + 0.094D_{12}^-$

subject to

$-x_1 + 0.214x_2 - 1.214x_3 + D_{11}^- - D_{11}^+ = 0.5,$

$0.294x_1 - 1.029x_2 - 0.931x_3 + D_{12}^- - D_{12}^+ = 2.891,$

$-2x_1 - x_2 - x_3 + D_{11}^- \leq 1,$

$-x_2 - x_3 + D_{12}^- \leq 3,$

$3x_1 + 5x_2 + x_3 \leq 35,$

$2x_1 - x_2 + 12x_3 \leq 20,$

$5x_2 + 6x_3 \leq 16$,

$x_1, x_2, x_3, D_{11}^-, D_{11}^+, D_{12}^-, D_{12}^+ \geq 0$

Using Lingo programming, the compromise solution of the 1st level problem is obtained as; $(x_1^0, x_2^0, x_3^0) = (0,2.3364,0)$.

4.2 Solving the 2nd level FGP model

$$min\, Z = 0.714D_{11}^{-} + 0.094D_{12}^{-} + 0.243D_{21}^{-} + 0.625D_{22}^{-}$$

$$subject\ to$$

$$-x_1 + 0.214x_2 - 1.214x_3 + D_{11}^{-} - D_{11}^{+} = 0.5,$$

$$0.294x_1 - 1.029x_2 - 0.931x_2 + D_{12}^{-} - D_{12}^{+} = 2.891,$$

$$-0.728x_1 + 1.215x_2 + 0.001x_3 + D_{21}^{-} - D_{21}^{+} = 3.641,$$

$$-1.25x_1 - 1.875x_2 - 2.25x_3 + D_{22}^{-} - D_{22}^{+} = 0.375,$$

$$-2x_1 - x_2 - x_3 + D_{11}^{-} \leq 1,$$

$$-x_2 - x_3 + D_{12}^{-} \leq 3,$$

$$-x_1 - x_3 + D_{21}^{-} \leq 4,$$

$$-2x_1 - x_3 + D_{22}^{-} \leq 6,$$

$$3x_1 + 5x_2 + x_3 \leq 35,$$

$$2x_1 - x_2 + 12x_3 \leq 20,$$

$$5x_2 + 6x_3 \leq 16\ ,$$

$$x_1 = 0,$$

$$x_2, x_3, D_{11}^{-}, D_{11}^{+}, D_{12}^{-}, D_{12}^{+}, D_{21}^{-}, D_{21}^{+}, D_{22}^{-}, D_{22}^{+} \geq 0.$$

Using Lingo programming, the compromise solution of the 2nd level problem is obtained as: $(\square_1^0, \square_2^0, \square_3^0) = (0,0,0)$.

4.3 Solving the 3rd level FGP model

$$min\, Z = 0.714D_{11}^{-} + 0.094D_{12}^{-} + 0.243D_{21}^{-} + 0.625D_{22}^{-} + 0.086D_{31}^{-} + 0.37D_{32}^{-}$$

$$subject\ to$$

$$-x_1 + 0.214x_2 - 1.214x_3 + D_{11}^{-} - D_{11}^{+} = 0.5,$$

$$0.294x_1 - 1.029x_2 - 0.931x_3 + D_{12}^{-} - D_{12}^{+} = 2.891,$$

$$-0.728x_1 + 1.215x_2 + 0.001x_3 + D_{21}^{-} - D_{21}^{+} = 3.641,$$

$$-1.25x_1 - 1.875x_2 - 2.25x_3 + D_{22}^{-} - D_{22}^{+} = 0.375,$$

$0.172x_1 - 0.172x_2 - 0.948x_3 + D_{31}^{-} - D_{31}^{+} = 1.724,$

$0.665x_1 - 2.815x_2 - 1.555x_3 + D_{12}^{-} - D_{12}^{+} = 6.075,$

$-2x_1 - x_2 - x_3 + D_{11}^{-} \leq 1,$

$-x_2 - x_3 + D_{12}^{-} \leq 3,$

$-x_1 - x_3 + D_{21}^{-} \leq 4,$

$-2x_1 - x_3 + D_{22}^{-} \leq 6,$

$-x_3 + D_{31}^{-} \leq 2,$

$-x_1 - 3x_2 - x_3 + D_{32}^{-} \leq 7,$

$3x_1 + 5x_2 + x_3 \leq 35,$

$2x_1 - x_2 + 12x_3 \leq 20,$

$5x_2 + 6x_3 \leq 16\,,$

$x_1 = 0,$

$x_2 = 0,$

$x_3, D_{11}^{-}, D_{11}^{+}, D_{12}^{-}, D_{12}^{+}, D_{21}^{-}, D_{21}^{+}, D_{22}^{-}, D_{22}^{+} \geq 0.$

Using Lingo programming, the compromise solution of the 3rd level problem is obtained as: $(x_1^0, x_2^0, x_3^0) = (0,0,0)$.

and $f_{11} = 2,\ \ f_{12} = 0.33333,\ \ f_{21} = 1.25,\ \ f_{22} = 0.5,\ \ f_{31} = 2,\ \ f_{32} = 0.85714$.

Similarly, applying the proposed algorithm to solve (FP2), (FP3) and (FP4), we get the following intervals:

The surely optimal range	The possibly optimal range
FLDM:	
$\left[\underline{f}_{11}^{L}, \underline{f}_{11}^{U}\right] = [2,3],$	$\left[\overline{f}_{11}^{L}, \overline{f}_{11}^{U}\right] = [1.1838462,4].$
$\left[\underline{f}_{12}^{L}, \underline{f}_{12}^{U}\right] = [0.333333,1],$	$\left[\overline{f}_{12}^{L}, \overline{f}_{12}^{U}\right] = [-0.0577434, 1.666666].$
SLDM:	
$\left[\underline{f}_{21}^{L}, \underline{f}_{21}^{U}\right] = [1.25,1.5],$	$\left[\overline{f}_{21}^{L}, \overline{f}_{21}^{U}\right] = [0.88788465, 1.75],$
$\left[\underline{f}_{22}^{L}, \underline{f}_{22}^{U}\right] = [0.5,0.666666],$	$\left[\overline{f}_{22}^{L}, \overline{f}_{22}^{U}\right] = [0.2720512667\ , 1].$
TLDM:	
$\left[\underline{f}_{31}^{L}, \underline{f}_{31}^{U}\right] = [2,2.5],$	$\left[\overline{f}_{31}^{L}, \overline{f}_{31}^{U}\right] = [1.3161538,3].$
$\left[\underline{f}_{32}^{L}, \underline{f}_{32}^{U}\right] = [0.58714,1],$	$\left[\overline{f}_{32}^{L}, \overline{f}_{32}^{U}\right] = [0.7108077816\ , 1.142857143].$

5 Conclusion and Summary

Multi-level multi-objective fractional programming problem (ML-MOFP) was considered where some or all of its coefficients in the objective function are rough intervals. Two FP problems with interval coefficients constructed. One of these problems was a FP where all of its coefficients are lower approximation of the rough intervals and the other problem was a FP where all of its coefficients are upper approximations of rough intervals. A fuzzy goal programming model has been formulated to obtain the satisfactory solution of the multi-level multi-objective fractional programming problem.

At the end, there exist many other open points for future work and research which should be explored and studied in the area of multi- level multi-objective rough interval optimization such as:

1. An algorithm is required for treating multi-level multi-objective integer fractional decision-making problems with rough parameters in the objective functions; in the constraints and in both.
2. An algorithm is needed for dealing with multi- level multi-objective mixed integer fractional decision-making problems with rough parameters in the objective functions; in the constraints and in both.
3. An algorithm must be investigated for treating multi- level multi-objective integer quadratic decision-making problems with rough parameters in the objective functions; in the constraints and in both.

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Competing Interests

Authors have declared that no competing interests exist.

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